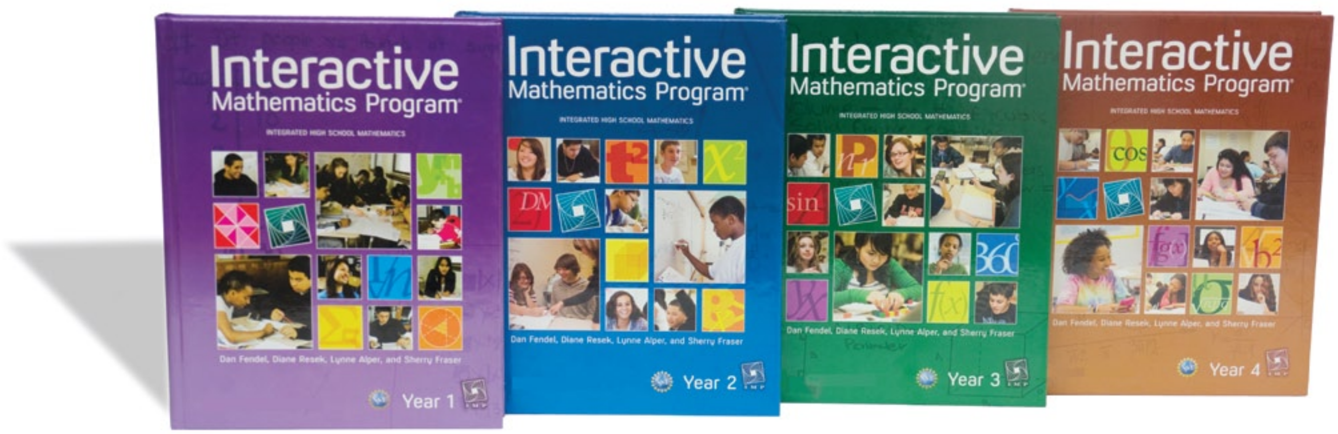
IMP SAMPLER

Interactive Mathematics Program



IT's ABOUT TIME®

Contents

About IMP

The Interactive Mathematics Program is a problem-based curriculum that offers challenging content and emphasizes mathematical reasoning. Designed and field-tested with support from the National Science Foundation, IMP has demonstrated in schools throughout the country that the successful study of advanced mathematics is an achievable standard for all students.

The benefits of IMP

- IMP's integrated, problem-based curriculum helps you teach challenging content in an approachable way that leads to student success. IMP has demonstrated in schools throughout the United States and Canada that learning advanced mathematics is an achievable standard for all students.

- IMP's stimulating problems and projects will help you teach your students to think creatively and critically. You can be sure that they will develop strong mathematical reasoning skills and the ability to use multiple strategies to solve problems.

- You'll be able to keep students interested and engaged with intriguing problems based on real-world situations, as well as fanciful scenarios.

- You will see your students become better mathematics communicators as they present their solutions and mathematical reasoning to small groups and to the entire class.

- You will be able to easily differentiate instruction as students work through hands-on activities.

- IMP's in-depth, over-time approach allows you to consistently prepare students for high-stakes exams.

- Doing stimulating, problem-based work inspires IMP students to take more years of college-prep mathematics than non-IMP students.

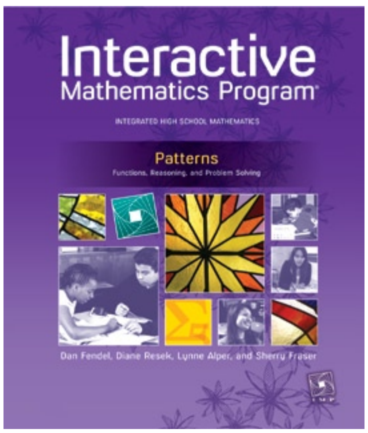
IMP Options

IMP is now available in three formats, to fit your needs:

- Four-year integrated sequence, covering Common Core State Standards for Mathematics (CCSSM) and much more
- Traditional Pathway Algebra 1-Geometry-Algebra 2 sequence, aligned to CCSSM content recommended by PARCC and the CCSSM Appendix A
- Individual units, for integrating problem-based learning and CCSSM Standards for Mathematical Practice into your curriculum.

Integrated Pathway and Individual Units

These are the units as they are organized in the integrated sequence. Each of these units is available for purchase as an individual unit book as well.



Year 1

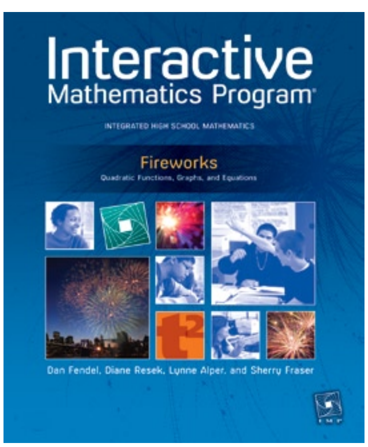
PATTERNS Students develop basic ideas about functions, integers, angles, and polygons. They learn how to work on mathematical investigations and report on their ideas both orally and in writing.

THE GAME OF PIG Students develop a mathematical analysis for a complex game based on an area model for probability.

THE OVERLAND TRAIL Students look at mid-19th-century Western migration in terms of the many linear relationships involved.

THE PIT AND THE PENDULUM Exploring an excerpt from this Edgar Allan Poe classic, students use data from experiments and statistical ideas, such as standard deviation, to develop a formula for the period of a pendulum.

SHADOWS Students use principles about similar triangles and basic trigonometry to develop formulas for finding the length of a shadow.



Year 2

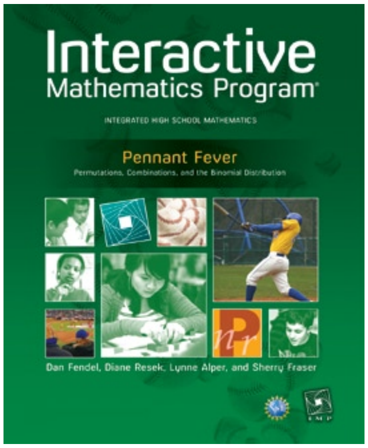
DO BEES BUILD IT BEST? Students study surface area, volume, and trigonometry to answer the question, "What is the best shape for a honeycomb?"

COOKIES In their work to maximize profits for a bakery, students deepen their understanding of the relationship between equations and inequalities and their graphs.

IS THERE REALLY A DIFFERENCE? Students build on prior experience with statistical ideas from IMP Year 1, expanding their understanding of statistical analysis.

FIREWORKS The central problem of this unit involves sending up a rocket to create a fireworks display. This unit builds on the algebraic investigations of Year 1, with a special focus on quadratic expressions, equations, and functions.

ALL ABOUT ALICE The unit starts with a model based on Lewis Carroll's *Alice's Adventures in Wonderland*, through which students develop the basic principles for working with exponents.



Year 3

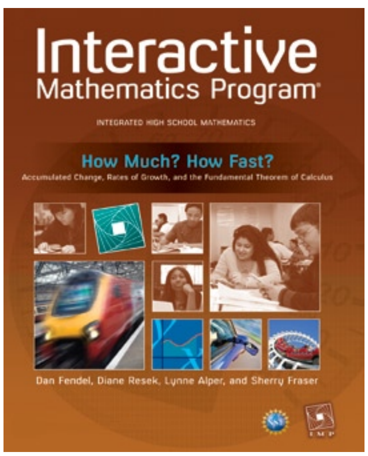
ORCHARD HIDEOUT Students study circles and coordinate geometry to determine how long it will take before the trees in a circular orchard grow so large that someone standing at the center of the orchard cannot see out.

MEADOWS OR MALLS? This unit concerns making a decision about land use and builds on skills learned in Cookies about graphing systems of linear inequalities and solving systems of linear equations.

SMALL WORLD, ISN'T IT? Beginning with a table of population data, students study situations involving rates of growth, develop the concept of slope, and then generalize this to the idea of the derivative.

PENNANT FEVER Students use combinatorics to develop the binomial distribution and find the probability that the team leading in the pennant race will ultimately win the pennant.

HIGH DIVE Using trigonometry, polar coordinates, and the physics of falling objects, students model this problem: When should a diver on a Ferris wheel aiming for a moving tub of water be released in order to create a splash instead of a splat?



Year 4

THE DIVER RETURNS This unit builds upon Year 3's High Dive problem: "When should a diver on a Ferris wheel aiming for a moving tub of water be released in order to create a splash instead of a splat?" In Year 4, students use vectors modeling horizontal and vertical components of the diver's initial velocity.

AS THE CUBE TURNS Students study the fundamental geometric transformations—translations, rotations, and reflections—in two and three dimensions, in order to create a display of a cube rotating around an axis in three-dimensional space.

THE WORLD OF FUNCTIONS In this unit, students explore families of functions in terms of various representations—tables, graphs, algebraic representations, and situations they can model. They also explore ways of combining functions using arithmetic operations and composition.

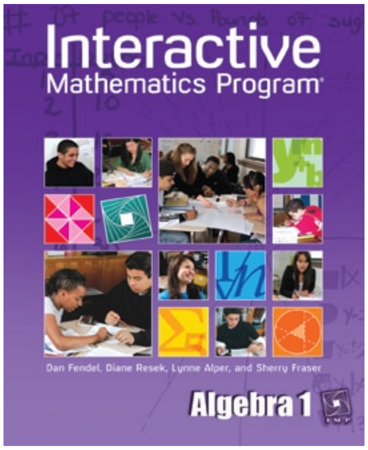
THE POLLSTER'S DILEMMA The central problem of this unit concerns an election poll, and students use normal distributions and standard deviations to find confidence intervals and see how concepts such as margin of error are used in polling results.

HOW MUCH? HOW FAST? This unit adds integrals to the derivative concepts explored in Year 3. Students solve accumulation problems using a version of the Fundamental Theorem of Calculus. They find that the derivative of the function that describes the amount of accumulation up to a particular time is the rate of accumulation, and that the function describing accumulation is an anti-derivative of the function describing the rate of accumulation.

Traditional Pathway

The original Interactive Mathematics Program's integrated units have been reordered and enhanced to provide a Traditional Pathway approach to the Common Core State Standards, aligned to PARCC Model Content Framework assessment standards. The units have been reworked, supplemented, and an entirely new unit has been added.

Algebra 1



THE OVERLAND TRAIL—VARIABLES, GRAPHS, LINEAR FUNCTIONS, AND EQUATIONS Students look at mid-19th-century Western migration in terms of the many linear relationships involved.

- Covers topics from the Number and Quantity strand related to units.
- Covers topics from the Algebra strand related to linear expressions, equations, and inequalities.
- Covers topics from the Functions strand related to modeling with linear functions.
- Covers topics from the Statistics and Probability strand related to representing, interpreting, and modeling two-variable data.

ALL ABOUT ALICE—EXPONENTS AND LOGARITHMS The unit starts with a model based on Lewis Carroll's *Alice's Adventures in Wonderland*, through which students develop the basic principles for working with exponents.

- Covers topics from the Number and Quantity strand related to real numbers.
- Covers topics from the Algebra strand related to exponents and exponential functions.
- Covers topics from the Functions strand related to modeling with exponential functions.

THE PIT AND THE PENDULUM—STANDARD DEVIATION AND CURVE FITTING Exploring an excerpt from this Edgar Allan Poe classic, students use data from experiments and statistical ideas, such as standard deviation, to develop a formula for the period of a pendulum.

- Covers topics from the Number and Quantity strand related to measurement accuracy.
- Covers topics from the Algebra strand related to modeling with equations.
- Covers topics from the Functions strand related to modeling.
- Covers topics from the Statistics and Probability strand related to representing, interpreting, and modeling one-variable and two-variable data.

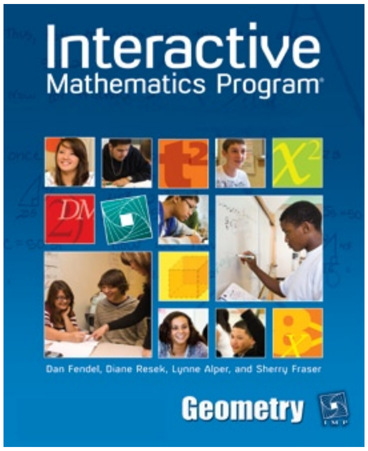
COOKIES—SYSTEMS OF EQUATIONS AND LINEAR PROGRAMMING In their work to maximize profits for a bakery, students deepen their understanding of the relationship between equations and inequalities and their graphs.

- Covers topics from the Algebra strand related to systems of equations.

FIREWORKS—QUADRATIC FUNCTIONS, GRAPHS, AND EQUATIONS The central problem of this unit involves sending up a rocket to create a fireworks display. This unit builds on the algebraic investigations of the earlier units, with a special focus on quadratic expressions, equations, and functions.

- Covers topics from the Algebra strand related to quadratic equations and functions.
- Covers topics from the Functions strand related to modeling with quadratic functions.

Geometry



SHADOWS—SIMILAR TRIANGLES AND PROPORTIONAL REASONING Students use principles about similar triangles and basic trigonometry to develop formulas for finding the length of a shadow.

- Covers topics from the Geometry strand related to similarity and trigonometry.

GEOMETRY BY DESIGN—TRANSFORMATIONS, CONSTRUCTION, AND PROOF Students explore the history of geometry and human design to learn the concepts of congruence, transformations, geometric construction, and proof.

- Covers topics from the Geometry strand related to congruence, transformations, construction, and proof.

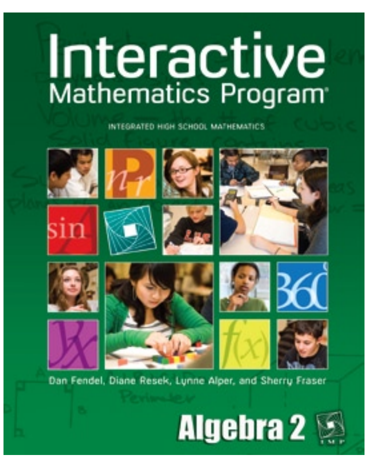
DO BEES BUILD IT BEST?—AREA, VOLUME, AND THE PYTHAGOREAN THEOREM Students study surface area, volume, and trigonometry to answer the question, "What is the best shape for a honeycomb?"

- Covers topics from the Geometry strand related area, volume, modeling, and solving problems with the Pythagorean theorem and the Laws of Sines and Cosines.

ORCHARD HIDEOUT—CIRCLES AND COORDINATE GEOMETRY Students study circles and coordinate geometry to determine how long it will take before the trees in a circular orchard grow so large that someone standing at the center of the orchard cannot see out.

- Covers topics from the Geometry strand related to construction, circles, coordinate geometry, and modeling.

Algebra 2



IMP Algebra 2 is in development.

Sample Content from Cookies—Systems of Equations and Linear Programming

Cookies is a typical IMP unit, which starts with a unit problem.

In the pages that follow, we've excerpted the first activity cluster (pages 114–124), in which students develop understanding of constraints and modeling with inequalities. In subsequent activities (not included here), students learn to graph inequalities, recognize a feasible region in a linear programming problem, solve systems of equations, and solve linear programming problems. Students use this new knowledge to solve the unit problem (pages 160–162). They then write their own linear programming problems, and culminate, as always, with compiling a portfolio to summarize their learning (pages 175–176).

Teacher's Guides discuss in detail how to guide students in their learning, including important questions to address in class discussion. A sample for the first activity of this unit, *How Many of Each Kind?*, is provided.

The In-Class and Take-Home Assessments, provided, show what students have learned.

Cookies

Systems of Equations and Linear Programming

Cookies—Systems of Equations and Linear Programming

Cookies and Inequalities

The central problem of this unit involves helping a bakery to maximize its profits. The problem is complex. First you will organize all of the information and express the bakery's situation in algebraic terms, using linear inequalities and linear expressions.

Sonia Mena creates a graph to help solve the bakery problem.

Group Activity

How Many of Each Kind?

Abby and Bing Woo own a small bakery that specializes in cookies. They make only two kinds of cookies—plain and iced. They need to decide *how many dozens* of each kind of cookie to make for tomorrow.

The Woos know that each dozen *plain* cookies requires 1 pound of cookie dough (and no icing). Each dozen *iced* cookies requires 0.7 pound of cookie dough and 0.4 pound of icing. They also know that each dozen plain cookies requires about 0.1 hour of preparation time. Each dozen iced cookies requires about 0.15 hour of preparation time. Finally, they know that no matter how many of each kind they make, they will sell them all.

Three factors limit the Woos' decision.

- The ingredients available: They have 110 pounds of cookie dough and 32 pounds of icing.
- The oven space available: They have room to bake a total of 140 dozen cookies.
- The preparation time available: Together they have 15 hours for cookie preparation.

continued

Why on earth should the Woos care how many cookies of each kind they make? Well, you guessed it! They want to make as much profit as possible. Plain cookies sell for $6.00 a dozen and cost $4.50 a dozen to make. Iced cookies sell for $7.00 a dozen and cost $5.00 a dozen to make.

The big question is

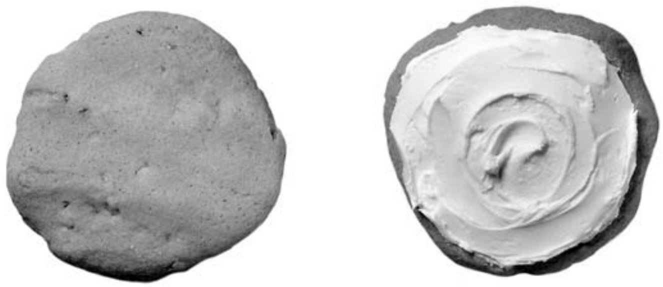
> *How many dozens of each kind of cookie should Abby and Bing make so their profit is as high as possible?*

1. Begin to answer the big question.
   a. Find one combination of dozens of plain cookies and dozens of iced cookies that will satisfy all of the conditions in the problem.
   b. Figure out how much profit the Woos will make on that combination of cookies.
2. Now find a different combination of dozens of cookies that fits the conditions but yields a greater profit.

Adapted from *Introduction to Linear Programming*, 2nd ed., by R. Stansbury Stockton (Boston: Allyn and Bacon, 1963).

A Simpler Cookie

The Woos have a rather complicated problem to solve. Let's make it simpler. Finding a solution to a simpler problem may lead to a method for solving the original problem.



Assume the Woos still make both plain and iced cookies and have 15 hours for cookie preparation. But now assume they have unlimited amounts of cookie dough and icing. Also assume that they have an unlimited amount of oven space.

The other information is unchanged.

- Preparing a dozen plain cookies requires 0.1 hour.
- Preparing a dozen iced cookies requires 0.15 hour.
- Plain cookies sell for $6.00 a dozen.
- Plain cookies cost $4.50 a dozen to make.
- Iced cookies sell for $7.00 a dozen.
- Iced cookies cost $5.00 a dozen to make.

As before, the Woos know that no matter how many of each kind of cookie they make, they will sell them all.

1. Find at least five combinations of plain and iced cookies that the Woos could make without working more than 15 hours. For each combination, compute the profit.
2. Find the combination of plain and iced cookies that you think would give the Woos the most profit. Explain why you think no other combination will yield a greater profit.

Investigating Inequalities

Part I: Manipulating Inequalities

You have learned about ways to change equations so they still hold true. For instance, suppose you have a true equation—that is, two expressions that are equal. You could add the same quantity to both sides of the equation, and the resulting expressions would still be equal.

For example, the statement $3 + 8 = 5 + 6$ is true, because $3 + 8$ and $5 + 6$ are both equal to 11. If you add 7 to both sides, the resulting statement is $3 + 8 + 7 = 5 + 6 + 7$. This statement is also true.

1. Investigate whether similar principles hold true for inequalities. The inequality $4 > 3$ is true. Starting with this inequality, perform each of these operations and examine whether the resulting statement is true.

   - Add the same number to both sides of the inequality.
   - Subtract the same number from both sides of the inequality.
   - Multiply both sides of the inequality by the same number.
   - Divide both sides of the inequality by the same number.

   For example, if you multiply both sides of the inequality $4 > 3$ by 2, the statement becomes $4 \cdot 2 > 3 \cdot 2$. Your task for each operation is to determine whether the new statement is true no matter what "the same number" is.

   Try different possibilities for "the same number." Use both positive and negative values.

2. After you finish investigating the inequality $4 > 3$, try a different true inequality. See whether you reach the same conclusions.

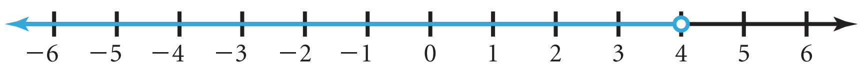
3. When you are done exploring, state your conclusions. Make them as general as possible.

continued

Part II: Graphing Inequalities

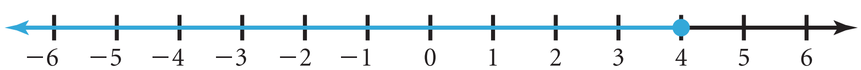
If an inequality contains a single variable, you can picture all the numbers that make the inequality true by shading them on a number line. This is called the graph of the **inequality.** An inequality using $<$ or $>$ is called a *strict inequality*. An inequality using $\leq$ or $\geq$ is called a *nonstrict inequality*.

For example, the colored portion of this number line represents the graph of the strict inequality $x < 4$.

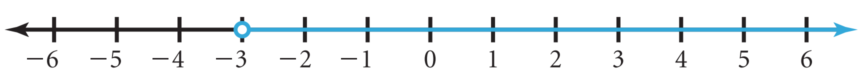


The open circle at the number 4 on the number line means that the number 4 is not included in the graph. (The number 4 is not included because substituting 4 for x gives a false statement.)

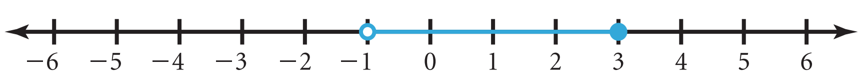
If you want to include a particular number as part of the graph, you mark that point with a filled-in circle. For example, the colored portion of the next diagram represents the graph of the nonstrict inequality $x \leq 4$.



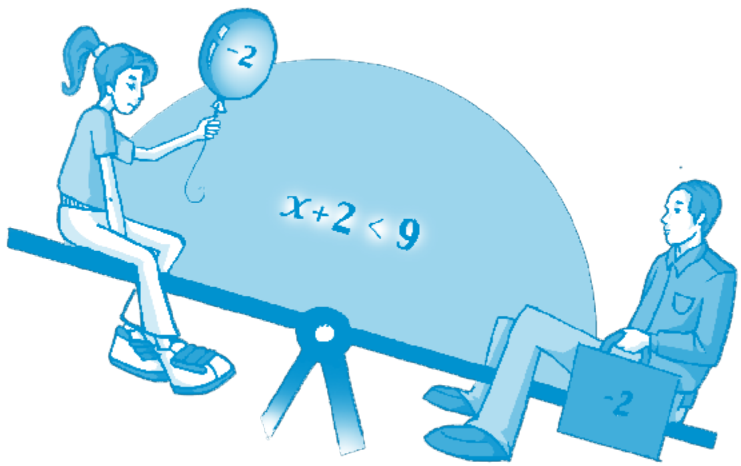
4. Draw the graph of the inequality $x > -2$.
5. Draw the graph of the inequality $x \leq 0$.
6. What inequality goes with this graph?



7. How would you use inequalities to describe this graph?



My Simplest Inequality



In *Investigating Inequalities,* you started with a true inequality. You explored which operations you could perform on both sides to create another true inequality.

When inequalities involve variables, you want to know if the operation produces an **equivalent inequality.** As with equations, two inequalities are equivalent if any number that makes one of them true also makes the other true.

For example, the inequalities $x + 2 < 9$ and $2x + 4 < 18$ are equivalent. The numbers that make both inequalities true are precisely the numbers less than 7. For instance, substituting 5 for x makes both statements true. Substituting 10 for x makes both false. That is, $5 + 2 < 9$ and $2 \cdot 5 + 4 < 18$ are true, while $10 + 2 < 9$ and $2 \cdot 10 + 4 < 18$ are false.

Part I: One Variable Only

If an inequality has only one variable, you can often find an equivalent inequality that essentially gives the solution. For instance, by subtracting 2 from both sides of $x + 2 < 9$, you get the equivalent inequality $x < 7$. This tells you that the solutions to $x + 2 < 9$ are the numbers less than 7 (and only those numbers).

continued

1. For each inequality, perform operations to get equivalent inequalities until you obtain one that shows the solution.
   a. $2x + 5 < 8$
   b. $3x - 2 \geq x + 1$
   c. $3x + 7 \leq 5x - 9$
   d. $4 - 2x > 7 + x$

Part II: Two or More Variables

When an inequality has more than one variable, you can't put it into a form that directly describes the solution. But you can often write the inequality in a simpler, equivalent form by combining terms.

For example, suppose you start with the inequality

$$9x - 4y - 2 \geq 3x + 10y + 6$$

These steps will produce a sequence of simpler, equivalent inequalities.

$9x - 2 \geq 3x + 14y + 6$ (adding $4y$ to both sides)

$6x - 2 \geq 14y + 6$ (subtracting $3x$ from both sides)

$6x \geq 14y + 8$ (adding 2 to both sides)

All the **coefficients** in $6x \geq 14y + 8$ are even. So, you can divide both sides of the inequality by 2. This gives $3x \geq 7y + 4$.

2. Each inequality in the sequence is equivalent to the original inequality. However, $3x \geq 7y + 4$ seems to be the simplest of all.
   a. Find numbers for x and y that fit the inequality $3x \geq 7y + 4$.
   b. Substitute the numbers that you found in part a into the original inequality, $9x - 4y - 2 \geq 3x + 10y + 6$. Verify that these numbers make the inequality true.
   c. Find numbers for x and y that do not fit the inequality $3x \geq 7y + 4$.

continued

d. Substitute the numbers you found in part c into the original inequality, $9x - 4y - 2 \geq 3x + 10y + 6$. Verify that these numbers make the inequality false.

e. Explain why your work in parts a to d is not enough to prove that the two inequalities are equivalent.

3. For each inequality, perform the appropriate operations to get simpler, equivalent inequalities.

a. $x + 2y > 3x + y + 2$

b. $\frac{x}{2} - y \leq 3x + 1$

c. $0.2y + 1.4x < 10$

Activity

Simplifying Cookies

As you have seen, you can express the **constraints** in the unit problem as inequalities using two variables. Suppose you use P to represent the number of dozens of plain cookies and I to represent the number of dozens of iced cookies. One way to write these inequalities is

$P + 0.7I \leq 110$ (for the amount of cookie dough)

$0.4I \leq 32$ (for the amount of icing)

$P + I \leq 140$ (for the amount of oven space)

$0.1P + 0.15I \leq 15$ (for the amount of preparation time)

1. Find at least one equivalent inequality for each of these "cookie inequalities." If possible, find one that you think is simpler than the given inequality.
2. For each of the original inequalities, do these things.
   a. Find a number pair for P and I that fits the inequality. Also find a number pair that does not fit the inequality.
   b. Verify that the number pair that fits the inequality also fits any equivalent inequalities you found.
   c. Verify that the number pair that does not fit the inequality also does not fit any of the equivalent inequalities you found.

Cookies and the University

You're now ready to solve the unit problem. You will use the feasible region, a family of parallel **profit lines,** and a pair of linear equations to find the cookie combination the Woos should make.

When you're done, you will apply your skills to a completely new problem about college admissions.

Moses Lazo, Anandika Muni, Mandy Mazik, and Vanessa Morales work together to solve the unit problem.

How Many of Each Kind? Revisited

Remember the activity *How Many of Each Kind?* Abby and Bing Woo have a small bakery, and they bake two kinds of cookies—plain cookies and iced cookies. Here is a summary of the key information about the situation.

Summary of the Situation

Facts

- Each dozen plain cookies requires 1 pound of cookie dough.
- Each dozen iced cookies requires 0.7 pound of cookie dough and 0.4 pound of icing.
- Each dozen plain cookies requires 0.1 hour of preparation time.
- Each dozen iced cookies requires 0.15 hour of preparation time.

Constraints

- The Woos have 110 pounds of cookie dough and 32 pounds of icing.
- The Woos have oven space to bake a total of 140 dozen cookies.
- The Woos have 15 hours available to prepare cookies.

Costs, Prices, and Sales

- Plain cookies cost $4.50 a dozen to make. They sell for $6.00 a dozen.
- Iced cookies cost $5.00 a dozen to make. They sell for $7.00 a dozen.
- No matter how many cookies of each kind they make, the Woos will sell them all.

The big question is

> *How many dozens of each kind of cookie should Abby and Bing make so that their profit is as high as possible?*

continued

Your Task

Imagine your group is a business consulting team. The Woos have come to you for help. Of course you want to give them the right answer. But you also want to explain clearly how you know you have found the best possible answer.

You may want to review what you already know. Look back at your notes and your earlier work on this problem. Then write a report for the Woos. Your report should contain these items.

- An answer to the Woos' dilemma. Your answer should include a summary of how much cookie dough, icing, and preparation time they will need. Tell them how many dozens of each kind of cookie they will need to bake. Also tell them how much profit they can expect.
- An explanation that will convince the Woos that your answer will give them the greatest profit.
- Any graphs, charts, equations, or diagrams necessary to support your explanation.

When you write your report, assume the Woos do not know the techniques you have learned in this unit about solving this type of problem.

Cookies Portfolio

Now you will put together your portfolio for *Cookies*. Do these three tasks.

- Write a cover letter that summarizes the unit.
- Choose papers to include from your work in the unit.
- Discuss your personal growth during the unit.

Cover Letter

Review the *Cookies* unit. Describe the unit's central problem and main mathematical ideas. Your description should give an overview of how the key ideas were developed. Also describe how you used them to solve the central problem.

continued

Selecting Papers

Your portfolio for *Cookies* should contain these things.

- *Beginning Portfolio Selection* and *Continued Portfolio Selection*

  Include the activities from the unit that you selected in *Beginning Portfolio Selection* and *Continued Portfolio Selection,* along with your written work on these activities.
- A Problem of the Week

  Select one of the three POWs you completed during this unit: *A Hat of a Different Color, Kick It!,* or *Shuttling Around.*
- *A Reflection on Money*
- *Get the Point*
- *"How Many of Each Kind?" Revisited*
- *"Producing Programming Problems" Write-up*

Personal Growth

Your cover letter should describe how the mathematical ideas were developed in the unit. As part of your portfolio, write about your own personal growth during this unit. You may want to address this question: *How do you think you have improved in your ability to make presentations?*

Also include any other thoughts you wish to share with a reader of your portfolio.

How Many of Each Kind?

Intent

Students are introduced to the unit problem and explore the situation informally.

Mathematics

In the unit problem, students deal with a set of constraints on ingredients, oven space, time, and cost as they try to maximize the profit from sales of two kinds of cookies. Each of the constraints can be expressed as a linear inequality in two variables, and the profit is a linear function of those same two variables. In this activity, students look for specific numeric examples that fit the constraints, and then they determine the profit for each example. They do this work without a formal introduction to linear programming, instead using their prior knowledge to follow the constraints introduced in the story.

Progression

Students work in groups to explore the central unit problem. They post their findings for later reference. In a class discussion, they develop inequalities to represent the constraints and a symbolic expression for the profit.

Approximate Time

70 minutes

Classroom Organization

Groups and whole class

Doing the Activity

Have students read the activity, perhaps allowing several students to read portions of it aloud. Then have students work in groups on the questions.

Part of the challenge of this problem is keeping track of all the numbers. Let students develop their own ways of organizing the information; one of the goals of the unit is to find ways to improve on their initial methods.

As you observe, make sure students realize that the numbers they are looking for are in *dozens*—for example, “4 dozen plain, 3 dozen iced,” not “48 plain, 36 iced.”

Students may at first think they have to make use of all the ingredients, oven space, and preparation time available. Let them become aware on their own that this is not required and is, in fact, impossible. You might ask such questions as, **Could the Woos make 1 dozen of each kind? What about 3 dozen plain and 5 dozen iced?**

Discussing and Debriefing the Activity

To prepare for a discussion, begin a chart like the following one that includes group number or name, dozens of plain cookies, dozens of iced cookies, dough used, icing used, time used, oven space used, and a profit column for plain cookies sold, iced cookies sold, and total profit. As groups find combinations for which they have enough ingredients, they can add their results to the chart even if they duplicate another group's combination. If that happens, ask the group to find and post an additional combination that works.

Group	Plain cookies (dozen)	Iced cookies (dozen)	Dough used	Icing used	Time used	Oven space used	Profit: plain cookies	Profit: iced cookies	Total profit

Have group representatives present their organizational schemes for keeping track of and computing the profit for various combinations of cookies. Then have representatives of other groups offer other possible combinations.

As combinations are suggested, ask students to check whether they satisfy the conditions by calculating the dough, icing, oven space, and preparation time required for that combination and determining whether the results fit the conditions. **Are you sure that this combination fits all the conditions? How do you know?**

Introduce the term **constraint** as a synonym for *condition.* You might remind students that they encountered this term when they were forming families in the Year 1 unit *The Overland Trail.*

Ask students how and when to compute the profit for each combination. They may recognize that it makes sense to wait until they can establish whether a combination fits all the constraints before they make the calculation.

It is just as important for students to understand why a combination is excluded as it is to show that it is included. They must also recognize that each condition is a separate constraint and that a combination must satisfy all four constraints. Ask, **What is a combination that does *not* fit all the constraints? Which constraint or constraints does it fail to satisfy?**

Constraints as Inequalities

Choose one of the conditions, such as the amount of dough available, to focus on and ask, **How did you decide whether a combination fits this constraint?** As a class, develop a statement that tells whether a combination fits that constraint; for example, *Take the number of dozens of iced cookies, multiply that by 0.7, and add the number of dozens of plain cookies. The result cannot be more than 110.*

You may have to start with a less-detailed statement and gradually get students to refine it. Questions such as **Do we want to multiply by the number of cookies?** can help prompt refinements such as talking about the number of *dozens* of cookies.

How could you express the "cookie dough constraint" symbolically? Suggest that students choose variables to represent "number of dozens of plain cookies" and "number of dozens of iced cookies." Using P and I for these two variables, for example, they should realize that the dough constraint can be expressed by the inequality $P + 0.7I \le 110$.

Then have students work in their groups to write verbal as well as symbolic statements for the other constraints. It is important that everyone be able to deal with both ways of expressing each condition. You may want to randomly ask groups to present verbal or symbolic expressions. The class should end up with a set of constraint inequalities that looks something like this:

$P + 0.7I \le 110$	(amount of cookie dough)
$0.4I \le 32$	(amount of icing)
$P + I \le 140$	(amount of oven space)
$0.1P + 0.15I \le 15$	(amount of preparation time)

Have students record these constraint inequalities on chart paper. Post the chart for later use, perhaps titling it "Cookie Constraints."

Have groups use the inequalities to check that the combinations discussed earlier really do satisfy the constraints. In doing so, students will be repeating their earlier computations. But they should go through the process at least once to confirm that the symbolic inequalities are saying the same thing as the verbal expressions of the conditions.

The Profit Expression

Now ask groups to develop a symbolic expression for the profit. If they have difficulty, have them examine the chart showing profit computed for various combinations. They should get the expression $1.5P + 2I$.

Students often make the mistake of treating the profit expression as another constraint. You may want to ask, **Why isn't the profit expression included in our list of constraints?**

Restrictions on P and I

Because of the problem context, P and I must be whole numbers. Discuss this important issue now if a student introduces it. Otherwise, you might ignore it for now, as the issue will be more engaging and meaningful for students if it arises initially with them or in the natural context of discussions of graphs or feasible regions later in the unit.

When the issue does come up, note that it has two aspects:

- Neither P nor I can be negative.

- P and I must be integers. (Allowing P and I to be multiples of $\frac{1}{12}$ would also make sense.)

If students raise either of these issues, you can broaden the discussion to include both, perhaps asking whether there are other restrictions on the "eligible" values for P and I. You might point out that the question of eligibility depends on the problem situation; negative or noninteger solutions make sense in some problems but not in others.

Concerning the restriction to nonnegative numbers, you might ask students if they can make this restriction by writing additional constraints—specifically, by expressing the condition of "not being negative" as inequalities. They should notice that adding the inequalities $P \geq 0$ and $I \geq 0$ to the constraints list fixes this aspect of the model. Write these inequalities on the posted constraint chart.

The issue of avoiding noninteger values is more complex, as it cannot be handled by additional inequalities. Perhaps the best approach is simply to explain that there is no easy way to handle it and that students should ignore it for now. Emphasize that this means they will need to be especially careful later on to check whether their solutions make sense. If it turns out that the solution that provides the maximum profit is somehow "ineligible," students will have to decide where to go from there. (It turns out in this problem that the combination with the maximum profit does have whole-number values for P and I.)

You can also take this opportunity to talk about mathematical modeling. Point out that we often need to simplify or ignore certain aspects of a problem in order to get a usable mathematical description. Reintroduce the term **mathematical model** for an abstract description of a real-world situation.

Finally, bring out that when simplifications are made, it becomes especially important to refer back to the original conditions of the problem after the mathematical analysis is completed.

Key Questions

Could the Woos make 1 dozen of each kind? What about 3 dozen plain and 5 dozen iced?

Are you sure that this combination fits all the conditions? How do you know?

What is a combination that does *not* fit all the constraints? Which constraint or constraints does it fail to satisfy?

How did you decide whether a combination fits this constraint?

How can you express this constraint symbolically?

Why isn't the profit expression included in our list of constraints?

In-Class Assessment

Part I: Graph It

Consider the following constraints.

$$x \geq y$$
$$x + y \geq 50$$
$$6x + 5y \leq 500$$
$$x \geq 0,\ y \geq 0$$

1. On graph paper, sketch the feasible region for this set of constraints.
2. Find the approximate coordinates of each vertex of the feasible region.

Part II: Solve It

Use algebra to solve each system of equations. Show and explain your work clearly.

3. $4x + 3y = 5$

   $2x - 5y = 9$

4. $4x - 6y = 20$

   $6x - 9y = 24$

Take-Home Assessment

Part I: What If . . . ?

These three problems are variations on the unit problem. Your task is to find the combination of plain and iced cookies that maximizes the Woos' profit in each new situation. Consider these three variations as three separate problems.

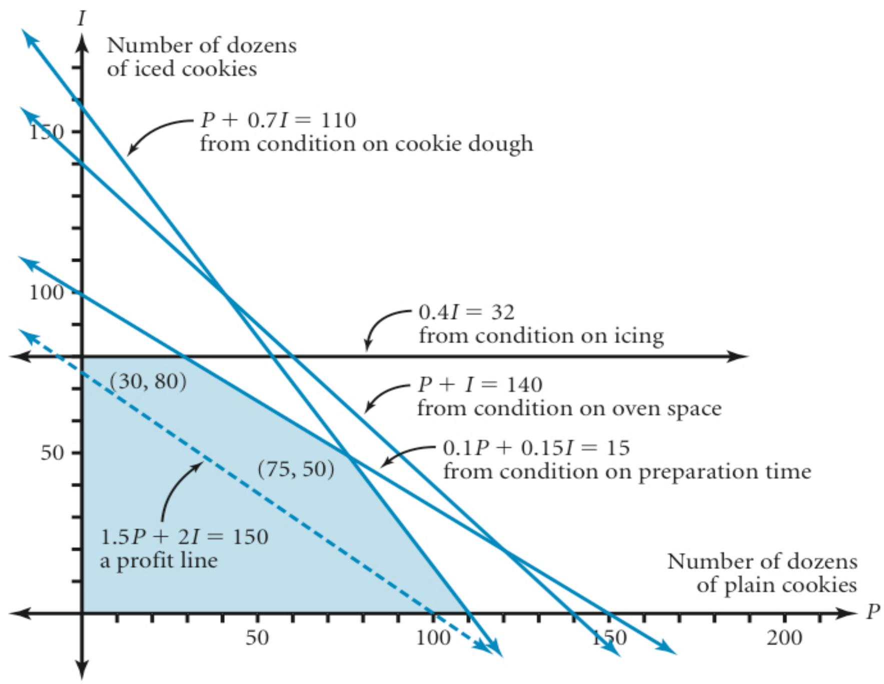
Each problem has a graph that shows the feasible region of the original problem. The shaded area represents that original feasible region.

Questions 1 and 2 show a profit line based on the original problem. Question 3 shows a profit line based on a different profit expression.

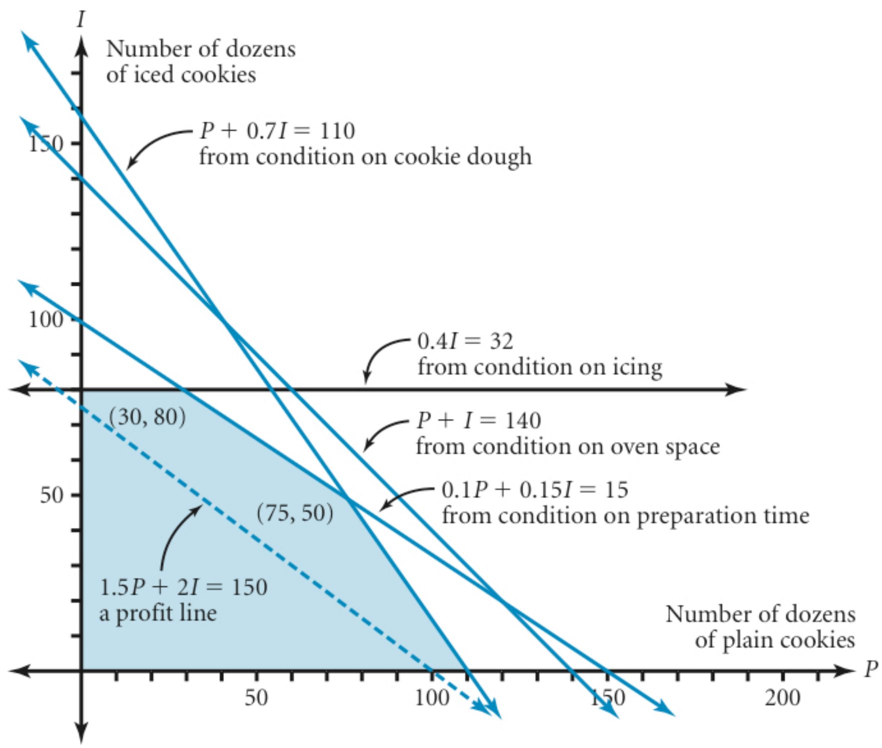
The other lines in each problem are the graphs of the original problem's constraint inequalities. Here are those inequalities.

$P + 0.7I \leq 110$	(for the amount of cookie dough)
$0.4I \leq 32$	(for the amount of icing)
$P + I \leq 140$	(for the amount of oven space)
$0.1P + 0.15I \leq 15$	(for the amount of the Woos' preparation time)

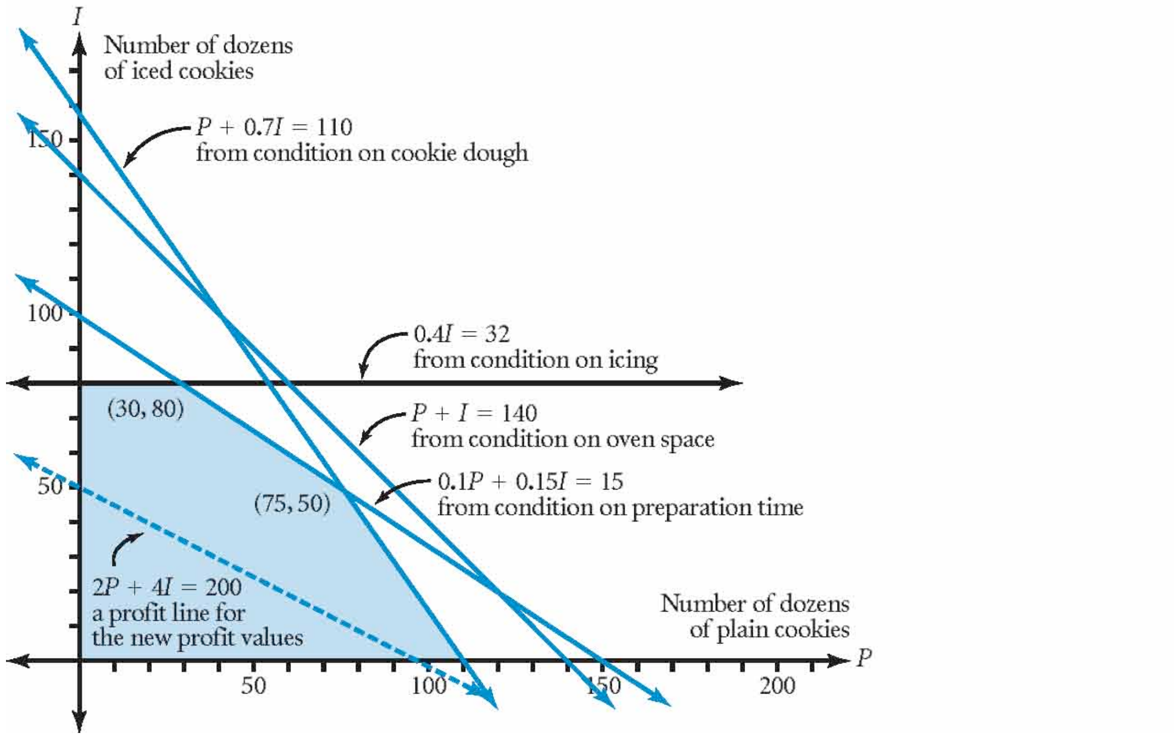
1. Suppose everything is the same as in the original problem, *except* that the Woos have an unlimited amount of dough. What combination of plain and iced cookies will maximize profit? Explain your answer.



2. Suppose everything is the same as in the original problem, *except* that the Woos have an additional constraint. They can't sell more than 60 dozen plain cookies. What combination of plain and iced cookies will maximize profit? Explain your answer.



3. Suppose everything is the same as in the original problem, *except* the profit on each kind of cookie. The Woos make a profit of $2.00 on each dozen plain cookies and $4.00 on each dozen iced cookies. (The original profits were $1.50 and $2.00.) What combination of plain and iced cookies will maximize profit? Explain your answer.



Part II: The Pebbles Rock at Big State U

The Rocking Pebbles are playing a concert at Big State University. The auditorium seats 2200 people. The concert manager decides to sell some tickets at $10 each and the rest at $15 each. How many of each kind should the manager sell if he wants ticket sales to total $26,600? Assume that all the tickets will be sold.

Find the answer by setting up and solving a system of two linear equations with two unknowns. Show and explain your work clearly.