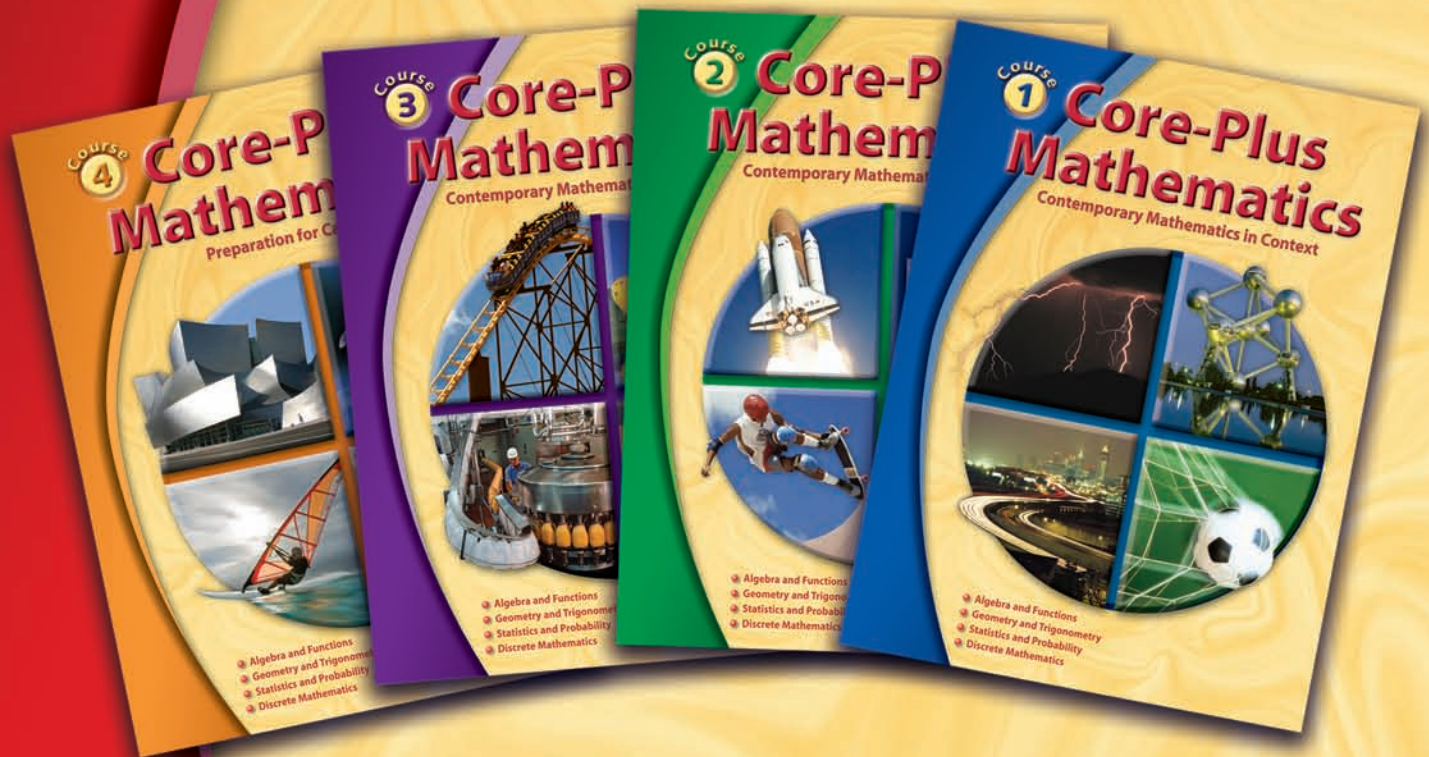


# Core-Plus Mathematics

Contemporary Mathematics in Context



## Scope and Sequence

- Algebra and Functions
- Geometry and Trigonometry
- Statistics and Probability
- Discrete Mathematics

# A Balanced and Unified Curriculum

*Core-Plus Mathematics* is a four-year unified curriculum that replaces the Algebra-Geometry-Advanced Algebra/Trigonometry-Precalculus sequence. Each course features interwoven strands of algebra and functions, geometry and trigonometry, statistics and probability, and discrete mathematics. Each of these strands is developed within coherent focused units connected by fundamental ideas such as symmetry, functions, matrices, and data analysis and curve-fitting. By actively investigating mathematics and its applications every year from an increasingly more mathematically sophisticated point of view, students' understanding of the mathematics in each strand deepens across the four-year curriculum. Mathematical connections between strands and ways of thinking mathematically that are common across strands are emphasized. These mathematical habits of mind include visual thinking, recursive thinking, searching for and explaining patterns, making and checking conjectures, reasoning with multiple representations, and providing convincing arguments and proofs.

## **Algebra and Functions**

The Algebra and Functions strand develops student ability to recognize, represent, and solve problems involving relations among quantitative variables. Central to the development is the use of functions as mathematical models. The key algebraic models in the curriculum are linear, exponential, power, polynomial, logarithmic, rational, and trigonometric functions. Modeling with systems of equations, both linear and nonlinear, is developed. Attention is also given to symbolic reasoning and manipulation.

## **Geometry and Trigonometry**

The primary goal of the Geometry and Trigonometry strand is to develop visual thinking and ability to construct, reason with, interpret, and apply mathematical models of patterns in visual and physical contexts. The focus is on describing patterns in shape, size, and location; representing patterns with drawings, coordinates, or vectors; predicting changes and invariants in shapes under transformations; and organizing geometric facts and relationships through deductive reasoning.

## **Statistics and Probability**

The primary role of the Statistics and Probability strand is to develop student ability to analyze data intelligently, to recognize and measure variation, and to understand the patterns that underlie probabilistic situations. The ultimate goal is for students to understand how inferences can be made about a population by looking at a random sample from that population. Graphical methods of data analysis, simulations, sampling, and experience with the collection and interpretation of real data are featured.

## **Discrete Mathematics**

The Discrete Mathematics strand develops student ability to solve problems using vertex-edge graphs, recursion, matrices, systematic counting methods (combinatorics), and mathematical methods for democratic decision making and information processing. Key themes are discrete mathematical modeling, optimization, and algorithmic problem solving.

# Organization of the Curriculum

The first three courses in the *Core-Plus Mathematics* series provide a significant core of broadly useful mathematics for all students. They were developed to prepare students for success in college, in careers, and in daily life in contemporary society. Course 4: Preparation for Calculus formalizes and extends the core program, with a focus on the mathematics needed to be successful in undergraduate programs requiring calculus. Unit titles for the four courses are given in the following table. Focus and content of these units are described on pages 3–7.

## Course 1

- 1 Patterns of Change
- 2 Patterns in Data
- 3 Linear Functions
- 4 Vertex-Edge Graphs
- 5 Exponential Functions
- 6 Patterns in Shape
- 7 Quadratic Functions
- 8 Patterns in Chance

## Course 2

- 1 Functions, Equations, and Systems
- 2 Matrix Methods
- 3 Coordinate Methods
- 4 Regression and Correlation
- 5 Nonlinear Functions and Equations
- 6 Network Optimization
- 7 Trigonometric Methods
- 8 Probability Distributions

## Course 3

- 1 Reasoning and Proof
- 2 Inequalities and Linear Programming
- 3 Similarity and Congruence
- 4 Samples and Variation
- 5 Polynomial and Rational Functions
- 6 Circles and Circular Functions
- 7 Recursion and Iteration
- 8 Inverse Functions

## Course 4: Preparation for Calculus

- 1 Families of Functions
- 2 Vectors and Motion
- 3 Algebraic Functions and Equations
- 4 Trigonometric Functions and Equations
- 5 Exponential Functions, Logarithms, and Data Modeling
- 6 Surfaces and Cross Sections
- 7 Concepts of Calculus
- 8 Counting Methods and Induction

# Core-Plus Mathematics 2nd Edition

## Course 1 Units

**Unit 1** **Patterns of Change** develops student ability to recognize and describe important patterns that relate quantitative variables, to use data tables, graphs, words, and symbols to represent the relationships, and to use reasoning and calculating tools to answer questions and solve problems.

*Topics include* variables and functions, algebraic expressions and recurrence relations, coordinate graphs, data tables and spreadsheets, and equations and inequalities.

**Unit 2** **Patterns in Data** develops student ability to make sense of real-world data through use of graphical displays, measures of center, and measures of variability.

*Topics include* distributions of data and their shapes, as displayed in dot plots, histograms, and box plots; measures of center including mean and median, and their properties; measures of variability including interquartile range and standard deviation, and their properties; and percentiles and outliers.

**Unit 3** **Linear Functions** develops student ability to recognize and represent linear relationships between variables and to use tables, graphs, and algebraic expressions for linear functions to solve problems in situations that involve constant rate of change or slope.

*Topics include* linear functions, slope of a line, rate of change, modeling linear data patterns, solving linear equations and inequalities, equivalent linear expressions.

**Unit 4** **Vertex-Edge Graphs** develops student understanding of vertex-edge graphs and ability to use these graphs to represent and solve problems involving paths, networks, and relationships among a finite number of elements, including finding efficient routes and avoiding conflicts.

*Topics include* vertex-edge graphs, mathematical modeling, optimization, algorithmic problem solving, Euler circuits and paths, matrix representation of graphs, vertex coloring and chromatic number.

**Unit 5** **Exponential Functions** develops student ability to recognize and represent exponential growth and decay patterns, to express those patterns in symbolic forms, to solve problems that involve exponential change, and to use properties of exponents to write expressions in equivalent forms.

*Topics include* exponential growth and decay functions, data modeling, growth and decay rates, half-life and doubling time, compound interest, and properties of exponents.

**Unit 6** **Patterns in Shape** develops student ability to visualize and describe two- and three-dimensional shapes, to represent them with drawings, to examine shape properties through both experimentation and careful reasoning, and to use those properties to solve problems.

*Topics include* Triangle Inequality, congruence conditions for triangles, special quadrilaterals and quadrilateral linkages, Pythagorean Theorem, properties of polygons, tilings of the plane, properties of polyhedra, and the Platonic solids.

**Unit 7** **Quadratic Functions** develops student ability to recognize and represent quadratic relations between variables using data tables, graphs, and symbolic formulas, to solve problems involving quadratic functions, and to express quadratic polynomials in equivalent factored and expanded forms.

*Topics include* quadratic functions and their graphs, applications to projectile motion and economic problems, expanding and factoring quadratic expressions, and solving quadratic equations by the quadratic formula and calculator approximation.

**Unit 8** **Patterns in Chance** develops student ability to solve problems involving chance by constructing sample spaces of equally-likely outcomes or geometric models and to approximate solutions to more complex probability problems by using simulation.

*Topics include* sample spaces, equally-likely outcomes, probability distributions, mutually exclusive (disjoint) events, Addition Rule, simulation, random digits, discrete and continuous random variables, Law of Large Numbers, and geometric probability.



## Course 2 Units

**Unit 1** **Functions, Equations, and Systems** reviews and extends student ability to recognize, describe, and use functional relationships among quantitative variables, with special emphasis on relationships that involve two or more independent variables.

*Topics include* direct and inverse variation and joint variation; power functions; linear equations in standard form; and systems of two linear equations with two variables, including solution by graphing, substitution, and elimination.

**Unit 2** **Matrix Methods** develops student understanding of matrices and ability to use matrices to represent and solve problems in a variety of real-world and mathematical settings.

*Topics include* constructing and interpreting matrices, row and column sums, matrix addition, scalar multiplication, matrix multiplication, powers of matrices, inverse matrices, properties of matrices, and using matrices to solve systems of linear equations.

**Unit 3** **Coordinate Methods** develops student understanding of coordinate methods for representing and analyzing properties of geometric shapes, for describing geometric change, and for producing animations.

*Topics include* representing two-dimensional figures and modeling situations with coordinates, including computer-generated graphics; distance in the coordinate plane, midpoint of a segment, and slope; coordinate and matrix models of rigid transformations (translations, rotations, and line reflections), of size transformations, and of similarity transformations; animation effects.

**Unit 4** **Regression and Correlation** develops student understanding of the characteristics and interpretation of the least squares regression equation and of the use of correlation to measure the strength of the linear association between two variables.

*Topics include* interpreting scatterplots; least squares regression, residuals and errors in prediction, sum of squared errors, influential points; Pearson's correlation coefficient and its properties, lurking variables, and cause and effect.

**Unit 5** **Nonlinear Functions and Equations** introduces function notation, reviews and extends student ability to construct and reason with functions that model parabolic shapes and other quadratic relationships in science and economics, with special emphasis on formal symbolic reasoning methods, and introduces common logarithms and algebraic methods for solving exponential equations.

*Topics include* formalization of function concept, notation, domain and range; factoring and expanding quadratic expressions, solving quadratic equations by factoring and the quadratic formula, applications to supply and demand, break-even analysis; common logarithms and solving exponential equations using base 10 logarithms.

**Unit 6** **Network Optimization** develops student understanding of vertex-edge graphs and ability to use these graphs to solve network optimization problems.

*Topics include* optimization, mathematical modeling, algorithmic problem solving, digraphs, trees, minimum spanning trees, distance matrices, Hamilton circuits and paths, the Traveling Salesperson Problem, critical paths, and the PERT technique.

**Unit 7** **Trigonometric Methods** develops student understanding of trigonometric functions and the ability to use trigonometric methods to solve triangulation and indirect measurement problems.

*Topics include* sine, cosine, and tangent functions of measures of angles in standard position in a coordinate plane and in a right triangle; indirect measurement; analysis of variable-sided triangle mechanisms; Law of Sines and Law of Cosines.

**Unit 8** **Probability Distributions** further develops student ability to understand and visualize situations involving chance by using simulation and mathematical analysis to construct probability distributions.

*Topics include* Multiplication Rule, independent and dependent events, conditional probability, probability distributions and their graphs, waiting-time (or geometric) distributions, expected value, and rare events.

## Course 3 Units

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|---------------|--|
| <b>Unit 1</b> | <p><b>Reasoning and Proof</b> develops student understanding of formal reasoning in geometric, algebraic, and statistical contexts and of basic principles that underlie those reasoning strategies.</p> <p><i>Topics include</i> inductive and deductive reasoning strategies; principles of logical reasoning—Affirming the Hypothesis and Chaining Implications; relation among angles formed by two intersecting lines or by two parallel lines and a transversal; rules for transforming algebraic expressions and equations; design of experiments including the role of randomization, control groups, and blinding; sampling distribution, randomization test, and statistical significance.</p> |
| <b>Unit 2</b> | <p><b>Inequalities and Linear Programming</b> develops student ability to reason both algebraically and graphically to solve inequalities in one and two variables, introduces systems of inequalities in two variables, and develops a strategy for optimizing a linear function in two variables within a system of linear constraints on those variables.</p> <p><i>Topics include</i> inequalities in one and two variables (including absolute value and quadratic inequalities), number line graphs, interval notation, systems of linear inequalities, and linear programming.</p>  |
| <b>Unit 3</b> | <p><b>Similarity and Congruence</b> extends student understanding of similarity and congruence and their ability to use those relations to solve problems and to prove geometric assertions with and without the use of coordinates.</p> <p><i>Topics include</i> connections between Law of Cosines, Law of Sines, and sufficient conditions for similarity and congruence of triangles, centers of triangles, applications of similarity and congruence in real-world contexts, necessary and sufficient conditions for parallelograms, sufficient conditions for congruence of parallelograms, and midpoint connector theorems.</p>   |
| <b>Unit 4</b> | <p><b>Samples and Variation</b> extends student understanding of the measurement of variability, develops student ability to use the normal distribution as a model of variation, introduces students to the binomial distribution and its use in decision making, and introduces students to the probability and statistical inference involved in control charts used in industry for statistical process control.</p> <p><i>Topics include</i> normal distribution, standardized scores, binomial distributions (shape, expected value, standard deviation), normal approximation to a binomial distribution, odds, statistical process control, control charts, and the Central Limit Theorem.</p>   |
| <b>Unit 5</b> | <p><b>Polynomial and Rational Functions</b> extends student ability to represent and draw inferences about polynomial and rational functions using symbolic expressions and manipulations.</p> <p><i>Topics include</i> definition and properties of polynomials, operations on polynomials; completing the square, proof of the quadratic formula, solving quadratic equations (including complex number solutions), vertex form of quadratic functions; definition and properties of rational functions, operations on rational expressions.</p>   |
| <b>Unit 6</b> | <p><b>Circles and Circular Functions</b> develops student understanding of relationships among special lines, segments, and angles in circles and the ability to use properties of circles to solve problems; develops student understanding of circular functions and the ability to use these functions to model periodic change; and extends student ability to reason deductively in geometric settings.</p> <p><i>Topics include</i> properties of chords, tangent lines, and central and inscribed angles of circles; linear and angular velocity; radian measure of angles; and circular functions as models of periodic change.</p>  |
| <b>Unit 7</b> | <p><b>Recursion and Iteration</b> extends student ability to represent, analyze, and solve problems in situations involving sequential and recursive change.</p> <p><i>Topics include</i> iteration and recursion as tools to model and analyze sequential change in real-world contexts, including compound interest and population growth; arithmetic, geometric, and other sequences; arithmetic and geometric series; finite differences; linear and nonlinear recurrence relations; and function iteration, including graphical iteration and fixed points.</p>   |
| <b>Unit 8</b> | <p><b>Inverse Functions</b> develops student understanding of inverses of functions with a focus on logarithmic functions and their use in modeling and analyzing problem situations and data patterns.</p> <p><i>Topics include</i> inverses of functions; logarithmic functions and their relation to exponential functions, properties of logarithms, equation solving with logarithms; and inverse trigonometric functions and their applications to solving trigonometric equations.</p>  |

## Course 4 Units: Preparation for Calculus

**Unit 1** **Families of Functions** extends student understanding of linear, exponential, quadratic, power, and trigonometric functions to model data patterns whose graphs are transformations of basic patterns; and develops understanding of operations on functions useful in representing and reasoning about quantitative relationships.

*Topics include* linear, exponential, quadratic, power, and trigonometric functions; data modeling; translation, reflection, and stretching of graphs; and addition, subtraction, multiplication, division, and composition of functions.

**Unit 2** **Vectors and Motion** develops student understanding of two-dimensional vectors and their use in modeling linear, circular, and other nonlinear motion.

*Topics include* concept of vector as a mathematical object used to model situations defined by magnitude and direction; equality of vectors, scalar multiples, opposite vectors, sum and difference vectors, dot product of two vectors, position vectors and coordinates; and parametric equations for motion along a line and for motion of projectiles and objects in circular and elliptical orbits.

**Unit 3** **Algebraic Functions and Equations** reviews and extends student understanding of properties of polynomial and rational functions and skills in manipulating algebraic expressions and solving polynomial and rational equations, and develops student understanding of complex number representations and operations.

*Topics include* polynomials, polynomial division, factor and remainder theorems, operations on complex numbers, representation of complex numbers as vectors, solution of polynomial equations, rational function graphs and asymptotes, and solution of rational equations and equations involving radical expressions.

**Unit 4** **Trigonometric Functions and Equations** extends student understanding of, and ability to reason with, trigonometric functions to prove or disprove two trigonometric expressions are identical and to solve trigonometric equations; to geometrically represent complex numbers and complex number operations and to find roots of complex numbers.

*Topics include* the tangent, cotangent, secant, and cosecant functions; fundamental trigonometric identities, sum and difference identities, double-angle identities; solving trigonometric equations and expression of periodic solutions; rectangular and polar representations of complex numbers, absolute value, DeMoivre's Theorem, and the roots of complex numbers.

**Unit 5** **Exponential Functions, Logarithms, and Data Modeling** extends student understanding of exponential and logarithmic functions to the case of natural exponential and logarithmic functions, solution of exponential growth and decay problems, and use of logarithms for linearization and modeling of data patterns.

*Topics include* exponential functions with rules in the form  $f(x) = Ae^{kx}$ , natural logarithm function, linearizing bivariate data and fitting models using log and log-log transformations.

**Unit 6** **Surfaces and Cross Sections** extends student ability to visualize and represent three-dimensional shapes using contours, cross sections, and reliefs, and to visualize and represent surfaces and conic sections defined by algebraic equations.

*Topics include* using contours to represent three-dimensional surfaces and developing contour maps from data; sketching surfaces from sets of cross sections; conics as planar sections of right circular cones and as locus of points in a plane; three-dimensional rectangular coordinate system; sketching surfaces using traces, intercepts and cross sections derived from algebraically-defined surfaces; and surfaces of revolution and cylindrical surfaces.

**Unit 7** **Concepts of Calculus** develops student understanding of fundamental calculus ideas through explorations in a variety of applied problem contexts and their representations in function tables and graphs.

*Topics include* instantaneous rates of change, linear approximation, area under a curve, and applications to problems in physics, business, and other disciplines.

**Unit 8** : **Counting Methods and Induction** extends student ability to count systematically and solve enumeration problems, and develops understanding of, and ability to do, proof by mathematical induction.

: *Topics include* systematic listing and counting, counting trees, the Multiplication Principle of Counting, Addition Principle of Counting, combinations, permutations, selections with repetition; the binomial theorem, Pascal's triangle, combinatorial reasoning; the general multiplication rule for probability; the Principle of Mathematical Induction; and the Least Number Principle.