CORE-PLUS MATHEMATICS PROJECT

COMMON CORE EDITION

IMPLEMENTING CORE-PLUS MATHEMATICS

Contemporary Mathematics in Context

McGraw Hill Education

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Core-Plus Mathematics
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As seen on page v, CPMP has been a collaborative effort that has drawn on the talents and energies of teams of mathematics educators at several institutions. This diversity of experiences and ideas has been a particular strength of the project. Special thanks is owed to the exceptionally capable support staff at these institutions, particularly to Angela Reiter, Hope Smith, Matthew Tuley, and Teresa Ziebarth at Western Michigan University.

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Our gratitude is expressed to the teachers and students in our 49 evaluation sites. The CCSS Edition of *Core-Plus Mathematics* builds on the strengths of the 1st and 2nd editions, which were shaped by multi-year field tests in 49 schools in Alaska, California, Colorado, Georgia, Idaho, Iowa, Kentucky, Michigan, Missouri, Ohio, South Carolina, Texas and Wisconsin. Each text is the product of a three-year cycle of research and development, pilot testing and refinement, and field testing and further refinement. Teachers' and students' experiences using the *Core-Plus Mathematics* units provided constructive feedback and suggested improvements that were immensely helpful.

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Introduction

The Core-Plus Mathematics Project (CPMP) was initially funded in 1992 by the National Science Foundation (NSF) to develop, evaluate, and in cooperation with a publisher, nationally disseminate a comprehensive high school mathematics curriculum. Over the last 20 plus years, the curriculum has been revised and updated in response to changing national standards for high school mathematics, reports from program users across the country and internationally, advances in technology, and what is known about student learning. The current edition strongly aligns with the *Common Core State Standards for Mathematics* (CCSS). For details, see the *CCSS Guide to Core-Plus Mathematics* available on ConnectED.

In 2010, NSF funds were awarded to a majority of the development team to create a separate fourth-year capstone course, *Transition to College Mathematics and Statistics*, intended for students planning to major in college programs or complete technical workforce programs that do *not* require calculus.

The curriculum development of *Core-Plus Mathematics* is similar in many respects to the iterative process outlined in the literature of design research (Design-Based Research Collective, 2003), design experiments (Brown, 1992; Collins, 1992), developmental research (Gravemeijer, 1994), and engineering research (Burkhardt & Schoenfeld, 2003). The CPMP development process over the last 20 years included iterative cycles of curriculum material design, development, field testing, evaluation, and revision including consultation with an international advisory board, mathematicians, instructional specialists with expertise in equity and access issues, and classroom teachers who have effectively taught the program for many years. Professional development support for field-test teachers and school districts adopting the program was viewed as an important responsibility of CPMP. Further articulation of the design and development of *Core-Plus Mathematics* is described in Fey and Hirsch (2007).

Core-Plus Mathematics is a problem-based, inquiry-oriented, technology-rich, four-year college-preparatory program that embodies the vision of high school mathematics portrayed in content, process, and teaching principles found in the National Council of Teachers of Mathematics *Standards* (NCTM, 1989, 1991, 1995, 2000); and more recently the mathematical practices and content expectations of the CCSS. Through investigations of real-life contexts, students develop a rich understanding of important mathematics that makes sense to them and that, in turn, enables them to make sense out of new situations and problems. In addition to the CCSS expectations, *Core-Plus Mathematics* aligns with curriculum, instruction, and assessment recommendations articulated in the following professional reports and documents:

- Strands of mathematical proficiency such as adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition, articulated in *Adding It Up: Helping Children Learn Mathematics* (National Research Council, 2001);
- *Curriculum Foundations Project: Voices of the Partner Disciplines* (Mathematical Association of America, 2004);

- *Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide* (Mathematical Association of America, 2004);
- *Guidelines for Assessment and Instruction in Statistics Education* (GAISE) Report (American Statistical Association, 2007);
- *Focus in High School Mathematics: Reasoning and Sense Making* (NCTM, 2009); and
- *Common Core State Standards for Mathematics* particularly the Standards for Mathematical Practice (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010).

In addition to the strong alignment with curricular recommendations in mathematics education such as the *Common Core State Standards*, *Core-Plus Mathematics* is designed to support the CCSS for English Language Arts & Literacy, particularly the anchor standards for reading, writing, speaking, and listening related to informational texts.

CCSS English Language Arts & Literacy
Literary Nonfiction and Historical, Scientific, and Technical Texts
Includes the subgenres of exposition, argument, and functional text in the form of personal essays, speeches, opinion pieces, essays about art or literature, biographies, memoirs, journalism, and historical, scientific, technical, or economic accounts (including digital sources) written for a broad audience

Each of the first three courses in *Core-Plus Mathematics* shares the following mathematical and instructional features.

An International-like Program

Each year, the curriculum advances students' understanding of mathematics along interwoven strands of algebra and functions, geometry and trigonometry, statistics and probability, and discrete mathematical modeling. These strands are unified by fundamental themes, by common topics, by mathematical practices, and by mathematical habits of the mind. Developing mathematics each year along multiple strands helps students develop a connected understanding of mathematics and nurtures their differing strengths and talents.

Mathematical Modeling

The problem-based curriculum emphasizes mathematical modeling including the processes of data collection, representation, interpretation, prediction, and simulation.

Access and Challenge

Rather than meaning that every student should receive identical instruction, equity demands that for every phase of the mathematics lesson, all students can participate substantially and not necessarily in the same way.

The curriculum is designed to make mathematics accessible to more students while at the same time challenging the most able students. Differences in student performance and interest can be accommodated by the depth and level of abstraction to which core topics are pursued, by the nature and degree of difficulty of applications, and by providing opportunities for student choice of homework tasks and projects.

Technology

Numeric, graphic, and symbolic manipulation capabilities such as those found in *CPMP-Tools*® and on many handheld graphing calculators are assumed and appropriately used throughout the curriculum. *CPMP-Tools* is a suite of software tools that provide powerful aids to learning mathematics and solving mathematical problems. (See pages 14–18 for further details.) This use of technology permits the curriculum and instruction to emphasize multiple linked representations (verbal, numerical, graphical, and symbolic) and to focus on goals in which mathematical thinking and problem solving are central.

Active Learning

Instructional materials promote active learning and teaching centered around collaborative investigations of problem situations followed by teacher-led whole-class summarizing activities that lead to analysis, abstraction, and further application of underlying mathematical ideas and principles. Students are actively engaged in exploring, conjecturing, verifying, generalizing, applying, proving, evaluating, and communicating mathematical ideas. (See snapshots of classrooms at: www.wmich.edu/cpmp/parentresource2/cpmpclassrooms.html)

Multi-dimensional Assessment

Comprehensive assessment of student understanding and progress through both curriculum-embedded formative assessment opportunities and summative assessment tasks supports instruction and enables monitoring and evaluation of each student's performance in terms of mathematical processes, content, and dispositions.

Overview of the Curriculum

Integrated Mathematics

Core-Plus Mathematics is an international-like curriculum that replaces an Algebra-Geometry-Advanced Algebra/Trigonometry-Precalculus sequence of high school mathematics courses. Each course features a coherent and connected development of important mathematics drawn from four strands: algebra and functions, geometry and trigonometry, statistics and probability, and discrete mathematical modeling.

Each of the strands is developed within focused units connected by fundamental ideas such as functions, matrices, symmetry, and data analysis and curve fitting. The strands also are connected across units by the CCSS Mathematical Practices.

These important mathematical practices include disposition toward, and proficiency in:

1. making sense of problems and persevering in solving them;
2. reasoning both quantitatively and algebraically;
3. constructing sound arguments and critiquing the reasoning of others;
4. using mathematics to model problems in everyday life, society, and in careers;

5. selecting and using appropriate tools, especially technological tools (graphing calculator, spreadsheet, computer algebra system, statistical packages, and dynamic geometry software);
6. communicating precisely and expressing calculations with an appropriate precision;
7. searching for and making use of patterns or structure in mathematical situations; and
8. identifying structure in repeated calculations, algebraic manipulation, and reasoning patterns.

Additionally, mathematical habits of mind such as visual thinking, recursive thinking, searching for and explaining patterns, making and checking conjectures, reasoning with multiple representations, describing and using algorithms, and providing convincing arguments and proofs are integral to each strand.

Important mathematical ideas are frequently revisited through attention to connections within and across strands and courses, enabling students to develop a robust and connected view and proficiency with mathematics.

Algebra and Functions

The *Algebra and Functions* strand develops student ability to recognize, represent, and solve problems involving relations among quantitative variables. Central to the development is the use of functions as mathematical models. The key algebraic models in the curriculum are linear, exponential, power, polynomial, logarithmic, rational, and trigonometric functions. Each algebraic model is investigated in at least four linked representations—verbal, graphic, numeric, and symbolic—with the aid of technology. Modeling with systems of equations, both linear and nonlinear, is developed. Attention is also given to symbolic reasoning and manipulation.

Geometry and Trigonometry

The primary goal of the *Geometry and Trigonometry* strand is to develop visual thinking and the ability to construct, reason with, interpret, and apply mathematical models of patterns in visual and physical contexts. The focus is on describing patterns with regard to shape, size, and location; representing patterns with drawings, coordinates, or vectors; predicting changes and invariants in figures under transformations; and organizing geometric facts and relationships through deductive reasoning.

Statistics and Probability

The primary role of the *Statistics and Probability* strand is to develop student ability to analyze data intelligently, to recognize and measure variation, and to understand the patterns that underlie probabilistic situations. The ultimate goal is for students to understand how inferences can be made about a population by looking at a sample from that population. Graphical methods of data analysis, simulations, sampling, and experience with the collection and interpretation of real data are featured.

Discrete Mathematical Modeling

The *Discrete Mathematics* strand develops student ability to model and solve problems using recursion, matrices, vertex-edge graphs, and systematic counting methods (combinatorics). Key themes are discrete mathematical modeling, optimization, and algorithmic problem solving.

Mathematical Modeling: The Core of *Core-Plus Mathematics*

Core-Plus Mathematics is designed so that students engage in the modeling process and thus engage in the mathematical behaviors identified in the CCSS Mathematical Practices as the primary vehicle for learning the mathematics and statistics elaborated in the CCSS content standards.

Prior to the era of the *Common Core State Standards for Mathematics*, many mathematics educators have encouraged learning mathematics through modeling of problems. Problems that engage students in mathematical modeling have the following features and benefits (Dewey, 1929, 1956; Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier, & Wearne, 1996):

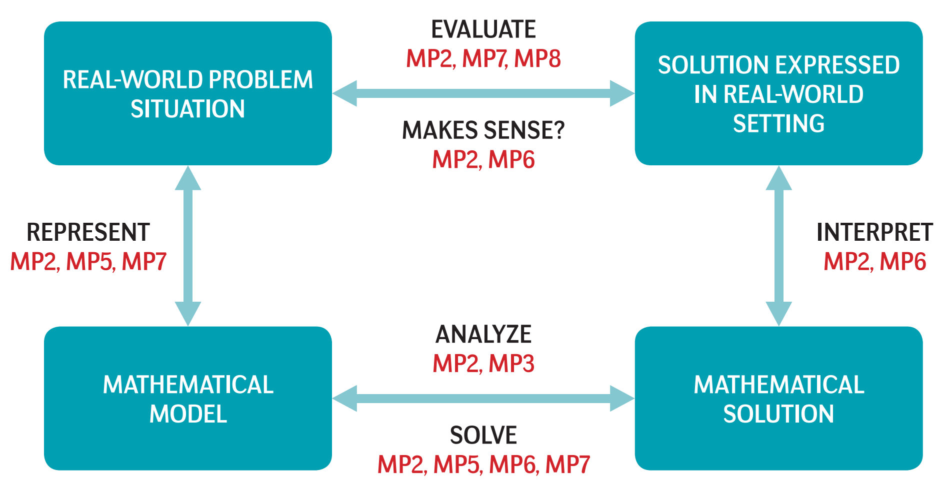
- Problems are identified in context.
- Problems are studied through active engagement.
- Conclusions are reached as problems are (at least partially) resolved.
- The benefits lie not only in the solutions to the problems, but the new relationships that are discovered.

Mathematical modeling in the CCSS for high school mathematics is a conceptual category and is only in the mathematical practices. Although standards are not listed for the mathematical modeling conceptual category, many of the content standards are designated as modeling standards under the other five conceptual categories.

The diagram below describes the modeling process and the mathematical practices engaged in during each phase of the process.

Process of Mathematical Modeling

Connecting Mathematical Practices (MP) and Content Standards (CS)
MP1 and MP4 are the overarching focal points of the entire process.



Courses 1–3

Each of the first three courses of *Core-Plus Mathematics* consists of eight units. Each unit contains two to four multi-day lessons in which major mathematics ideas are developed through investigations focused on sense-making and reasoning. Most investigations are developed from rich applied problems. Some are developed from examining mathematical patterns, structure, and procedures. Each unit also includes a "Looking Back" lesson to help students review and organize their thinking related to the mathematics learned in the unit. The time needed to complete a unit (in class periods of about 55 minutes) ranges from approximately two to six weeks. Unit titles for the three-year core curriculum are listed in the following table. Descriptions and mathematical topics for each of these units can be found on pages 8–10.

Unit Titles for Courses 1–3

Course 1		Course 2		Course 3	
UNIT 1	Patterns of Change	UNIT 1	Functions, Equations, and Systems	UNIT 1	Reasoning and Proof
UNIT 2	Patterns in Data	UNIT 2	Matrix Methods	UNIT 2	Inequalities and Linear Programming
UNIT 3	Linear Functions	UNIT 3	Coordinate Methods	UNIT 3	Similarity and Congruence
UNIT 4	Discrete Mathematical Modeling	UNIT 4	Regression and Correlation	UNIT 4	Samples and Variation
UNIT 5	Exponential Functions	UNIT 5	Nonlinear Functions and Equations	UNIT 5	Polynomial and Rational Functions
UNIT 6	Patterns in Shape	UNIT 6	Modeling and Optimization	UNIT 6	Circles and Circular Functions
UNIT 7	Quadratic Functions	UNIT 7	Trigonometric Methods	UNIT 7	Recursion and Iteration
UNIT 8	Patterns in Chance	UNIT 8	Probability Distributions	UNIT 8	Inverse Functions

The eight *Core-Plus Mathematics* units in each course should be taught in the order they have been developed to retain the learning progressions, coherence, and connections designed into the program. Although CPMP recognizes that deviations from the developed learning progressions within the program are made to meet student needs, careful attention must be given to any changes. In the front matter of each Teacher's Guide are two pathways for district consideration: a *Common Core State Standards* (CCSS) Pathway and a *Core-Plus Mathematics* program (CPMP) Pathway.

The organization of the student texts differs in several other ways from most conventional textbooks. Boxed-off definitions, "worked out" examples, and content summaries are not as prevalent as in conventional texts. Students learn mathematics by doing mathematics. Concept ideas are developed as students complete investigations; later, concept definitions are formalized. Mathematical ideas are developed and then shared by groups of students at strategically placed points within the lessons. These class discussions then lead to a class summary of shared understandings.

Course 4 Options

Course 4: Preparation for Calculus formalizes and extends the core program with a focus on the mathematics needed to be successful in STEM (science, technology, engineering, and mathematics) undergraduate programs. This course shares many of the design features of Courses 1–3. Course 4 unit titles are given in the following table. Descriptions and mathematical topics for each of these units can be found on page 11.

An alternative fourth-year course, *Transition to College Mathematics and Statistics* (TCMS), with similar design features as *Core-Plus Mathematics*, has been developed, field-tested in schools, and revised for publication under a separate NSF-funded grant. The TCMS course is designed as a capstone course for both conventional and integrated high school mathematics programs. Students who successfully complete three high school mathematics courses designed to meet the CCSS together with the proposed TCMS course will be well-prepared for two-year or four-year college programs that do not require calculus and also for apprentice programs leading to career-level jobs. TCMS course titles are given below. Descriptions and mathematics topics for the TCMS units can be found on page 12. See www.wmich.edu/tcms/ for more information on this program and your McGraw-Hill Education sales representative for sample materials.

Course 4 Options

Preparation for Calculus		***Transition to College Mathematics and Statistics***	
UNIT 1	Families of Functions	UNIT 1	Interpreting Categorical Data
UNIT 2	Vectors and Motion	UNIT 2	Functions Modeling Change
UNIT 3	Algebraic Functions and Equations	UNIT 3	Counting Methods
UNIT 4	Trigonometric Functions and Equations	UNIT 4	Mathematics of Financial Decision-Making
UNIT 5	Exponential Functions, Logarithms, and Data Modeling	UNIT 5	Binomial Distributions and Statistical Inference
UNIT 6	Surfaces and Cross Sections	UNIT 6	Informatics
UNIT 7	Concepts of Calculus	UNIT 7	Spatial Visualization and Representations
UNIT 8	Counting Methods and Induction	UNIT 8	Mathematics of Democratic Decision-Making

Course 1 Units and Descriptions

UNIT 1	**Patterns of Change** develops student ability to recognize and describe important patterns that relate quantitative variables, to use data tables, graphs, words, and symbols to represent the relationships, and to use reasoning and calculating tools to answer questions and solve problems. ***Topics include*** variables and functions, algebraic expressions and recurrence relations, coordinate graphs, data tables and spreadsheets, and equations and inequalities.
UNIT 2	**Patterns in Data** develops student ability to summarize, represent, and interpret real-world data on a single count or measurement variable through the use of graphical displays of the distribution, measures of center, and measures of spread. ***Topics include*** distributions of data and their shapes, as displayed in dot plots, histograms, and box plots; measures of center (mean and median) and their properties; measures of spread including interquartile range and standard deviation and their properties; and percentiles and outliers.
UNIT 3	**Linear Functions** develops student ability to recognize and represent linear relationships between variables and to use tables, graphs, and algebraic expressions for linear functions to solve problems in situations that involve constant rate of change or slope. ***Topics include*** linear functions, slope of a line, rate of change, modeling linear data patterns, solving linear equations and inequalities, equivalent linear expressions.
UNIT 4	**Discrete Mathematical Modeling** develops student ability in modeling, reasoning, and problem solving as they use vertex-edge graphs to model and solve problems about networks, paths, and relations. ***Topics include*** vertex-edge graphs, mathematical modeling, optimization, algorithmic problem solving, using Euler paths to find efficient routes, using vertex coloring to avoid conflicts, and matrix representation of graphs to aid interpretation.
UNIT 5	**Exponential Functions** develops student ability to recognize and represent exponential growth and decay patterns, to express those patterns in symbolic forms, to solve problems that involve exponential change, and to use properties of exponents to write expressions in equivalent forms. ***Topics include*** exponential growth and decay functions, data modeling, growth and decay rates, half-life and doubling time, compound interest, and properties of exponents.
UNIT 6	**Patterns in Shape** develops student ability to visualize and describe two- and three-dimensional shapes, to represent them with drawings, to examine shape properties through both experimentation and careful reasoning, and to use those properties to solve problems. ***Topics include*** Triangle Inequality, congruence conditions for triangles, special quadrilaterals and quadrilateral linkages, Pythagorean Theorem, properties of polygons, tilings of the plane, properties of polyhedra, cylinders, cones, and the Platonic solids.
UNIT 7	**Quadratic Functions** develops student ability to recognize and represent quadratic relations between variables using data tables, graphs, and symbolic formulas, to solve problems involving quadratic functions, and to express quadratic polynomials in equivalent factored and expanded forms. ***Topics include*** quadratic functions and their graphs, applications to projectile motion and economic problems, expanding and factoring quadratic expressions, and solving quadratic equations by the quadratic formula and calculator approximation.
UNIT 8	**Patterns in Chance** develops student ability to solve problems involving chance by constructing sample spaces of equally-likely outcomes or geometric models and to use simulation to decide whether a model is consistent with the data. ***Topics include*** sample spaces, equally-likely outcomes, probability distributions, mutually exclusive (disjoint) events, Addition Rule, union, intersection, two-way frequency tables, simulation, random digits, discrete and continuous random variables, Law of Large Numbers, and geometric probability.

Course 2 Units and Descriptions

UNIT 1	**Functions, Equations, and Systems** reviews and extends student ability to recognize, describe, and use functional relationships among quantitative variables, with special emphasis on relationships that involve two or more independent variables. ***Topics include*** direct and inverse variation and joint variation; power functions; linear equations in standard form; and systems of two linear equations with two variables, including solution by graphing, substitution, and elimination.
UNIT 2	**Matrix Methods** develops student understanding of matrices and ability to use matrices to model and solve problems in a variety of real-world and mathematical settings. ***Topics include*** constructing and interpreting matrices, matrix addition, scalar multiplication, matrix multiplication, powers of matrices, inverse matrices, comparing algebraic properties of matrices to those of real numbers, and using matrices to solve systems of linear equations.
UNIT 3	**Coordinate Methods** develops student understanding of coordinate methods for representing and analyzing properties of geometric shapes, for describing geometric change, and for producing animations. ***Topics include*** representing two-dimensional figures and modeling situations with coordinates, including computer-generated graphics; distance in the coordinate plane, midpoint of a segment, and slope; coordinate and matrix models of rigid transformations (translations, rotations, and line reflections), of size transformations, and of similarity transformations; animation effects.
UNIT 4	**Regression and Correlation** develops student ability to describe how two quantitative variables on a scatterplot are related, including fitting a function to the data and the use of correlation to measure the strength of a linear association between the two variables. ***Topics include*** construct and interpret scatterplots; compute and interpret a linear model including slope and intercept, residuals, and the correlation coefficient; sum of squared errors; influential points; and distinguish between correlation and causation.
UNIT 5	**Nonlinear Functions and Equations** introduces function notation, reviews and extends student ability to construct and reason with functions that model parabolic shapes and other quadratic relationships in science and economics, with special emphasis on formal symbolic reasoning methods, and introduces common logarithms and algebraic methods for solving exponential equations. ***Topics include*** formalization of function concept, notation, domain and range; factoring and expanding quadratic expressions, solving quadratic equations by factoring and the quadratic formula, applications to supply and demand, break-even analysis; common logarithms and solving exponential equations using base 10 logarithms.
UNIT 6	**Modeling and Optimization** develops student ability in mathematical modeling, optimization, and problem solving, through study of vertex-edge graphs, as students model and solve problems about networks, paths, and circuits. ***Topics include*** mathematical modeling, optimization, algorithmic problem solving, and using minimum spanning trees, Hamilton paths, the Traveling Salesperson Problem, critical paths, and the PERT technique to solve network optimization problems.
UNIT 7	**Trigonometric Methods** develops student understanding of trigonometric functions and the ability to use trigonometric methods to solve triangulation and indirect measurement problems. ***Topics include*** sine, cosine, and tangent functions of measures of angles in standard position in a coordinate plane and in a right triangle; indirect measurement; analysis of variable-sided triangle mechanisms; derivation and application of the Law of Sines and Law of Cosines.
UNIT 8	**Probability Distributions** develops student understanding of independent events, conditional probability, and expected value and how to use them to interpret data and evaluate outcomes of decisions. ***Topics include*** two-way frequency tables, Multiplication Rule, independent and dependent events, conditional probability, probability distributions and their graphs, waiting-time (or geometric) distributions, and expected value for games of chance and for applications such as insurance.

Course 3 Units and Descriptions

UNIT 1	**Reasoning and Proof** develops student understanding of formal reasoning in geometric, algebraic, and statistical contexts and of basic principles that underlie those reasoning strategies. ***Topics include*** inductive and deductive reasoning strategies; principles of logical reasoning—Affirming the Hypothesis and Chaining Implications; properties of line reflections; relation among angles formed by two intersecting lines or by two parallel lines and a transversal; rules for transforming algebraic expressions and equations; design of experiments; use of data from a randomized experiment to compare two treatments, sampling distribution constructed using simulation, randomization test, and statistical significance; inference from sample surveys, experiments, and observational studies and how randomization relates to each.
UNIT 2	**Inequalities and Linear Programming** develops student ability to reason both algebraically and graphically to solve inequalities in one and two variables, introduces systems of inequalities in two variables, and develops a strategy for optimizing a linear function in two variables within a system of linear constraints on those variables. ***Topics include*** inequalities in one and two variables, number line graphs, interval notation, systems of linear inequalities, and linear programming.
UNIT 3	**Similarity and Congruence** extends student understanding of similarity and congruence and their ability to use those relations to solve problems and to prove geometric assertions with and without the use of coordinates. ***Topics include*** connections between Law of Cosines, Law of Sines, and sufficient conditions for similarity and congruence of triangles; connections between transformations and sufficient conditions for congruence and similarity of triangles; centers of triangles, applications of similarity and congruence in real-world contexts; necessary and sufficient conditions for parallelograms, sufficient conditions for congruence of parallelograms, and midpoint connector theorems.
UNIT 4	**Samples and Variation** develops student ability to use the normal distribution as a model of variation, introduces students to the binomial distribution and its use in making inferences about population parameters based on a random sample, and introduces students to the probability and statistical inference used in industry for statistical process control. ***Topics include*** normal distribution, standardized scores and estimating population percentages, binomial distributions (shape, expected value, standard deviation), normal approximation to a binomial distribution, odds, statistical process control, and the Central Limit Theorem.
UNIT 5	**Polynomial and Rational Functions** extends student ability to represent and draw inferences about polynomial and rational functions using symbolic expressions and manipulations. ***Topics include*** definition and properties of polynomials, operations on polynomials; completing the square, proof of the quadratic formula, solving quadratic equations (including complex number solutions), vertex form of quadratic functions; definition and properties of rational functions, operations on rational expressions.
UNIT 6	**Circles and Circular Functions** develops student understanding of properties of special lines, segments, angles, and arcs in circles and the ability to use those properties to solve problems; develops student understanding of circular functions and the ability to use those functions to model periodic change; and extends student ability to reason deductively in geometric settings. ***Topics include*** properties of chords, tangent lines, and central and inscribed angles and their intercepted arcs; linear and angular velocity; radian measure of angles; and circular functions as models of periodic change.
UNIT 7	**Recursion and Iteration** extends student ability to model, analyze, and solve problems in situations involving sequential and recursive change. ***Topics include*** iteration and recursion as tools to model and solve problems about sequential change in real-world settings, including compound interest and population growth; arithmetic, geometric, and other sequences together with their connections to linear, exponential, and polynomial functions; arithmetic and geometric series; finite differences; linear and nonlinear recurrence relations; and function iteration, including graphical iteration and fixed points.
UNIT 8	**Inverse Functions** develops student understanding of inverses of functions with a focus on logarithmic functions and their use in modeling and analyzing problem situations and data patterns. ***Topics include*** inverses of functions; logarithmic functions and their relation to exponential functions, properties of logarithms, equation solving with logarithms; and inverse trigonometric functions and their applications to solving trigonometric equations.

Course 4: Preparation for Calculus Units and Descriptions	
UNIT 1	**Families of Functions** extends student understanding of linear, exponential, quadratic, power, and circular functions to model data patterns whose graphs are transformations of basic patterns; and develops understanding of operations on functions useful in representing and reasoning about quantitative relationships. ***Topics include*** linear, exponential, quadratic, power, and trigonometric functions; data modeling; translation, reflection, and stretching of graphs; and addition, subtraction, multiplication, division, and composition of functions.
UNIT 2	**Vectors and Motion** develops student understanding of two-dimensional vectors and their use in modeling linear, circular, and other nonlinear motion. ***Topics include*** concept of vector as a mathematical object used to model situations defined by magnitude and direction; equality of vectors, scalar multiples, opposite vectors, sum and difference vectors, dot product of two vectors, position vectors and coordinates; and parametric equations for motion along a line and for motion of projectiles and objects in circular and elliptical orbits.
UNIT 3	**Algebraic Functions and Equations** reviews and extends student understanding of properties of polynomial and rational functions and skills in manipulating algebraic expressions and solving polynomial and rational equations, and develops student understanding of complex number representations and operations. ***Topics include*** polynomials, polynomial division, factor and remainder theorems, operations on complex numbers, representation of complex numbers as vectors, solution of polynomial equations, rational function graphs and asymptotes, and solution of rational equations and equations involving radical expressions.
UNIT 4	**Trigonometric Functions and Equations** extends student understanding of, and ability to reason with, trigonometric functions to prove or disprove potential trigonometric identities and to solve trigonometric equations; develops student ability to geometrically represent complex numbers and their operations and to find powers and roots of complex numbers expressed in trigonometric form. ***Topics include*** fundamental trigonometric identities, sum and difference identities, double-angle identities; periodic solutions of trigonometric equations; definitions of secant, cosecant, and cotangent functions; absolute value and trigonometric form of complex numbers, De Moivre's Theorem, and roots of complex numbers.
UNIT 5	**Exponential Functions, Logarithms, and Data Modeling** extends student understanding of exponential and logarithmic functions to the case of natural exponential and logarithmic functions, solution of exponential growth and decay problems, and use of logarithms for linearization and modeling of data patterns. ***Topics include*** exponential functions with rules in the form $f(x) = Ae^{kx}$, natural logarithm function, linearizing bivariate data and fitting models using log and log-log transformations.
UNIT 6	**Surfaces and Cross Sections** extends student ability to visualize and represent three-dimensional shapes using contours, cross sections, and reliefs, and to visualize and represent surfaces and conic sections defined by algebraic equations. ***Topics include*** using contours to represent three-dimensional surfaces and developing contour maps from data; sketching surfaces from sets of cross sections; conics as planar sections of right circular cones and as loci of points in a plane; three-dimensional rectangular coordinate system; sketching surfaces using traces, intercepts and cross sections derived from algebraically-defined surfaces; and surfaces of revolution and cylindrical surfaces.
UNIT 7	**Concepts of Calculus** develops student understanding of fundamental calculus ideas through explorations in a variety of applied problem contexts and their representations in function tables and graphs. ***Topics include*** instantaneous rates of change, linear approximation, area under a curve, and applications to problems in physics, business, and other disciplines.
UNIT 8	**Counting Methods and Induction** extends student ability to count systematically and solve enumeration problems in a variety of real-world and mathematical settings, and develops understanding of, and ability to carry out, proofs by mathematical induction and by use of the Least Number Principle. ***Topics include*** systematic listing, counting trees, the Multiplication Principle of Counting, the Addition Principle of Counting, combinations, permutations, selections with repetition; the Binomial Theorem, Pascal's triangle, combinatorial reasoning; the General Multiplication Rule for Probability; proof by mathematical induction; and arguments using proof by contradiction and the Least Number Principle.

Transition to College Mathematics and Statistics Units and Descriptions	
UNIT 1	**Interpreting Categorical Data** develops student understanding of two-way frequency tables, conditional probability and independence, and using data from a randomized experiment to compare two treatments. ***Topics include*** two-way tables, graphical representations, comparison of proportions including absolute risk reduction and relative risk, characteristics and terminology of well-designed experiments, expected frequency, chi-square test of homogeneity, statistical significance.
UNIT 2	**Functions Modeling Change** extends student understanding of linear, exponential, quadratic, power, trigonometric, and logarithmic functions to model quantitative relationships and data patterns whose graphs are transformations of basic patterns. ***Topics include*** linear, exponential, quadratic, power, circular, and base-10 logarithmic functions; mathematical modeling; translation, reflection, stretching, and compressing of graphs with connections to symbolic forms of corresponding function rules.
UNIT 3	**Counting Methods** extends student ability to count systematically and solve enumeration problems using permutations and combinations. ***Topics include*** systematic listing and counting, counting trees, the Multiplication Principle of Counting, Addition Principle of Counting, combinations, permutations, selections with repetition; the binomial theorem, Pascal's triangle, combinatorial reasoning; and the general multiplication rule for probability.
UNIT 4	**Mathematics of Financial Decision-Making** extends student facility with the use of linear, exponential, and logarithmic functions, expressions, and equations in representing and reasoning about quantitative relationships, especially those involving financial mathematical models. ***Topics include*** forms of investment, simple and compound interest, future value of an increasing annuity, comparing investment options, continuous compounding and natural logarithms; amortization of loans and mortgages, present value of a decreasing annuity, and comparing auto loan and lease options.
UNIT 5	**Binomial Distributions and Statistical Inference** develops student understanding of the rules of probability; binomial distributions; expected value; testing a model; simulation; making inferences about the population based on a random sample; margin of error; and comparison of sample surveys, experiments, and observational studies and how randomization relates to each. ***Topics include*** review of basic rules and vocabulary of probability (addition and multiplication rules, independent events, mutually exclusive events); binomial probability formula; expected value; statistical significance and P value; design of sample surveys including random sampling and stratified random sampling; response bias; sample selection bias; sampling distribution; variability in sampling and sampling error; margin of error; and confidence interval.
UNIT 6	**Informatics** develops student understanding of the mathematical concepts and methods related to information processing, particularly on the Internet, focusing on the key issues of access, security, accuracy, and efficiency. ***Topics include*** elementary set theory and logic; modular arithmetic and number theory; secret codes, symmetric-key and public-key cryptosystems; error-detecting codes (including ZIP, UPC, and ISBN) and error-correcting codes (including Hamming distance); and trees and Huffman coding.
UNIT 7	**Spatial Visualization and Representations** extends student ability to visualize and represent three-dimensional shapes using contour diagrams, cross sections, and relief maps; to use coordinate methods for representing and analyzing three-dimensional shapes and their properties; and to use geometric and algebraic reasoning to solve systems of linear equations and inequalities in three variables and linear programming problems. ***Topics include*** using contours to represent three-dimensional surfaces and developing contour maps from data; sketching surfaces from sets of cross sections; three-dimensional rectangular coordinate system; sketching planes using traces, intercepts, and cross sections derived from algebraic representations; systems of linear equations and inequalities in three variables; and linear programming.
UNIT 8	**Mathematics of Democratic Decision-Making** develops student understanding of the mathematical concepts and methods useful in making decisions in a democratic society, as related to voting and fair division. ***Topics include*** preferential voting and associated vote-analysis methods such as majority, plurality, runoff, points-for-preferences (Borda method), pairwise-comparison (Condorcet method), and Arrow's theorem; weighted voting, including weight and power of a vote and the Banzhaf power index; and fair division techniques, including apportionment methods.

Technology

Core-Plus Mathematics is designed to take advantage of the power and potential of technology tools to support student learning and problem solving. Handheld graphing calculators, public domain computer software, and online resources are used to improve and enhance the teaching and learning of mathematics. Use of technology tools helps to facilitate mathematical curiosity, understanding, and sense-making that may otherwise be difficult or impossible without their use. For example, instructional time is gained as students efficiently generate, manipulate, and reason about a variety of representations (e.g., verbal, numerical, graphical, and algebraic) sometimes using built-in software data sets and examples.

Feedback from our work with schools teaching the first edition of *Core-Plus Mathematics* indicated that students had more access to computers outside of school than to handheld devices. This information and the recognition that students would be using a variety of computer software after graduation prompted the development of a suite of public domain computer software, *CPMP-Tools*, in tandem with curriculum development with specific applications in mind for the later editions.

The curriculum was developed with the vision reported in many recommendations supporting the use of technology in mathematics teaching and learning, such as found in the following professional documents and reports:

Jim Laser/CPMP

- "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning." (*Principles and Standards for School Mathematics*, NCTM, 2000, p. 24)
- "Technology is an essential tool for learning mathematics in the 21st century, and all schools must ensure that all their students have access to technology. Effective teachers maximize the potential of technology to develop students' understanding, stimulate their interest, and increase their proficiency in mathematics. When technology is used strategically, it can provide access to mathematics for all students." (*The Role of Technology in the Teaching and Learning of Mathematics*, 2008, p. 1)
- "Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. … They are able to use technological tools to explore and deepen their understanding of concepts." (*Common Core State Standards for Mathematics*, National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010, p. 7)
- "It is essential that teachers and students have regular access to technologies that support and advance mathematical sense making, reasoning, problem solving, and communications. Effective teachers optimize the potential of technology to develop students' understanding, stimulate their interest, and increase their proficiency in mathematics. When teachers use technology strategically, they can provide greater access to mathematics for all students." (*Technology in Teaching and Learning Mathematics*, NCTM, 2011, p. 1)

The curriculum assumes access to technology such as graphing calculators, computer algebra systems, data analysis tools, and dynamic geometry software. This may take the form of handheld technology such as graphing calculators. Technology tips are available in the *Unit Resource Masters* to assist students in learning to use Texas Instruments technology.

Computers and the *CPMP-Tools* Suite of Public Domain Software

Some schools provide student access to one-to-one computing for engaging in problem solving. Other schools use a combination of strategies to provide access to computers for students. Examples include access to one classroom computer for whole-class discussions, a set of four to six classroom computers for students to use as stations during group investigations, a portable set of laptops for individual or pairs of students, or occasional computer lab access for selective investigations.

The *CPMP-Tools* public domain software provides students access to powerful technology tools at school and at home. This accessibility offers flexible ways for students to develop and engage in mathematical reasoning and sense-making. Students and teachers can access *CPMP-Tools* online in mathematics classrooms, in school and local libraries, or any other place offering Internet access. In addition, it can be freely downloaded for offline use on school or home computers and is self-updating whenever connected to the Internet.

With the goal of reducing the steepness of the learning curve for students, the *CPMP-Tools* mathematical software and its functionality are organized by course to focus on the intended mathematics. Each of the four courses of *Core-Plus Mathematics* has a separate menu in *CPMP-Tools* offering access to specific sets of tools used during the course. To promote learning transfer from one tool, representation, or solution method to another, the linked set of tools share similar menu screens and, in some cases, interface with each other to provide smooth transitions between tools. The collective and linked nature of *CPMP-Tools* elevates the software from individual features for related mathematical tasks to a more coherent and integrated technology approach available for learning mathematics and solving problems.

This suite of Java-based mathematical software includes four families of programs.

Algebra and Functions—The software for work on algebra problems includes a spreadsheet and a computer algebra system (CAS) that produces tables and graphs of functions, manipulates algebraic expressions, and solves equations and inequalities.

Geometry and Trigonometry—The software for work on geometry problems includes an interactive drawing program for constructing, measuring, manipulating, transforming, and analyzing geometric figures in either a coordinate or coordinate-free environment.

Statistics and Probability—The software for work on data analysis and probability problems provides tools for graphic display and analysis of data, including finding function models for bivariate data and simulation of probability situations. It also includes a randomization distribution and distributions of sample means, medians, and standard deviations.

Discrete Mathematics—The software provides tools for constructing, manipulating, and analyzing discrete mathematical models.

In addition to the general-purpose software, custom apps have been developed for each mathematical strand. These custom apps were developed to allow exploration and analysis of specific mathematical or statistical concepts and topics. For example, students use custom apps to explore similar triangles, test congruence using geometric transformations, study binomial distributions and the randomization test, and analyze linear programming graphically.

CPMP-Tools includes files that provide electronic copies of most data sets essential for work on problems in each *Core-Plus Mathematics* course. When the opportunity to use computer tools for work on a particular investigation arises, students can select the *CPMP-Tools* menu corresponding to the content involved in the problem. Then they can select the submenu items corresponding to the required mathematical operations and data sets. (For example, consider the Compact Car problem that follows using a built-in data set.) Each unit overview in the *Teacher's Guide* provides general information related to *CPMP-Tools* use in the unit. Technology notes at point of use alert teachers to applicable software and specific data sets included in the software.

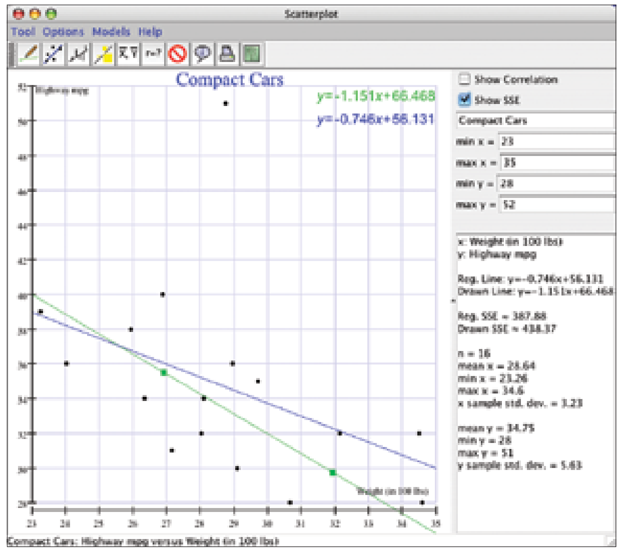
The availability of built-in editable data sets means that students can spend valuable class and assignment time on exploring, learning, and reasoning. In addition, the software provides save and print options. Students can begin solving a problem in class and continue outside of class by emailing partially completed work to themselves. They can also print their technology work and add additional text explanations to the page to complete the problem.

Sample Problems Using *CPMP-Tools* We encourage you to download and use the *CPMP-Tools* software from www.wmich.edu/cpmp/CPMP-Tools/. An extensive Help Menu explaining specific features has been developed. The material in this section provides examples of how software has been designed into the curriculum to support student learning and problem solving. Once students are familiar with the features of the software, they will have multiple addition tools to choose from as they solve problems. (CCSS MP5)

Sample 1

Course 2 Unit 4, Regression and Correlation, p. 287

2 Now, use data analysis software to examine the idea of finding the line on a scatterplot that minimizes the sum of squared errors (residuals), or SSE.



a. Using the moveable line capability of the software, visually find a line that you think best fits the compact car (*curb weight in hundreds of pounds*, *highway mpg*) data.

b. Compare the line you found visually and its equation with the regression line and its equation.

c. The Honda Civic Hybrid is an outlier and so may have a large effect on the location of the regression line. To investigate the effect of this point, first remove from the plot the lines you found in Part a.

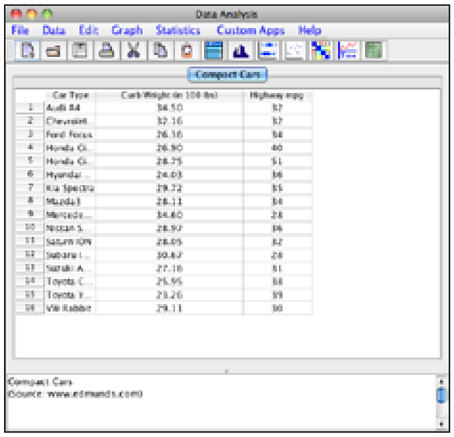
i. Delete the point for the Honda Civic Hybrid from the data set. How do the regression line and equation change?

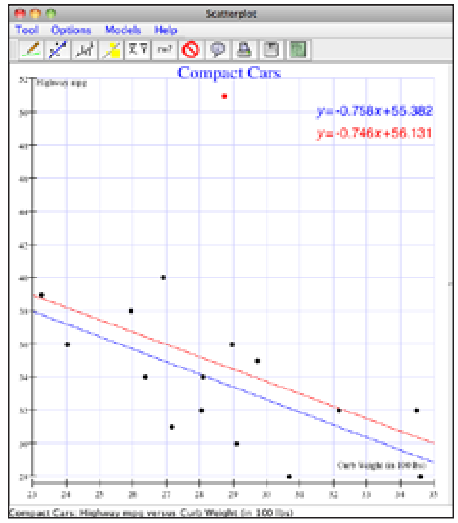
ii. Replace the point for the Honda Civic Hybrid and then delete the point for the Mercedes-Benz C280, which is the second heaviest car. How do the regression line and equation change in this case?

iii. Does the Honda Civic Hybrid or the Mercedes-Benz C280 have more influence on the regression line and equation?

In the problem above, students have the opportunity to display and analyze bivariate data from the built-in data set: Compact Cars. They connect algebra and statistics understanding by visually fitting a modeling line to data and comparing their model to the regression line and equation. Linked representations include numerical, symbolical, and graphical representations. The dynamic nature of the software quickly allows students to delete a point from the data set and explore and make sense of how the deletion of that point affects the visual representation and the algebraic representation of the model.

In the screen below, the linear equation for all the data is the red line and equation and the linear equation for the data with the Honda Civic Hybrid removed is the blue line and equation.





Sample 2

Course 3 Unit 3, Similarity and Congruence, p. 212

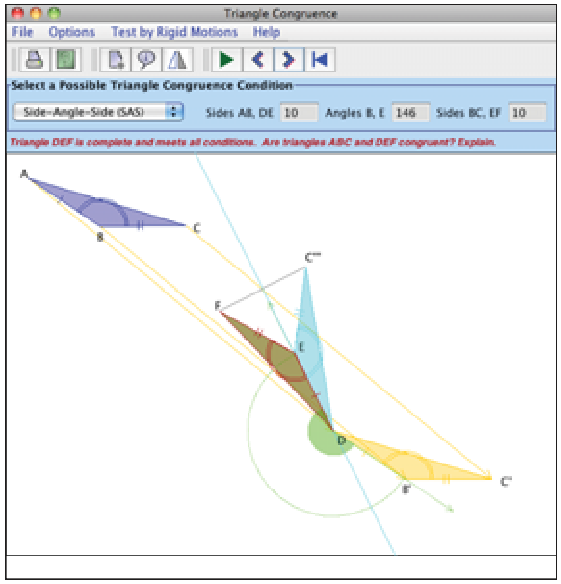
6 Using the "Transformations and Congruence" custom app in *CPMP-Tools*, explore how rigid transformations might be used to test the congruence of two triangles satisfying each condition below. For pairs of triangles that are shown to be congruent, identify the rigid transformation(s) that mapped the first triangle onto the second.

a. SSS Congruence Condition

b. SAS Congruence Condition

c. ASA Congruence Condition

In the above problem, students explore how one can test for congruence using rigid transformations prior to analyzing plans for deductive arguments in the student text. A sample display that students might produce during their exploration follows.



The dynamic exploratory work prior to examining plans for proofs will help students understand the plans for proofs and more effectively write their own deductive arguments.

Knowledge of the availability of features in the suite of tools offered by *CPMP-Tools* is an important prerequisite for making decisions about which tools are most appropriate for a given mathematical or statistical problem. To become proficient and strategic users of tools, students should regularly reflect on their use of tools and representations to consider the advantages and disadvantages of choices they have made. You will frequently find opportunities for reflection designed in the student materials. But other opportunities for student reflection are sure to arise as students use the tools in class and outside of class.

Curriculum Adoption: Advice and Tools

Recommendations From the Field

Curriculum coordinators and teachers who have engaged in mathematics curriculum study of *Core-Plus Mathematics* and other problem-based, student-centered programs recommend the following:

- Form a curriculum study committee to write mathematics program goals across grades K–12. Goal-writing committee members should include all high school mathematics teachers and administrative, counseling, and school board representatives. Once draft goals are written for K–12 mathematics, it is advisable to have wider discussion and input opportunities that involve teachers from partner disciplines and a broad spectrum of parents before the goals are finalized. Work for consensus, although there will likely be some different viewpoints expressed.

- Build understanding of your mathematics program goals and a support base from administrators, counselors, parents, board members, business/community leaders, and other departments within your high school.
- Use resources listed above that align with your K–12 mathematics goals or develop other resources to match, taking into consideration your state's mathematics education expectations.
- Order sample textbook materials and begin a review process that includes working many of the problems in the student materials.

Curriculum Analysis Tools

For the next few years, schools in many states will be transitioning to the *Common Core State Standards for Mathematics* K–12. If your district is considering adopting *Core-Plus Mathematics* as a response to the CCSS, you may wish to utilize the curriculum analysis tools developed by the Common Core State Standards Mathematics Curriculum Materials Analysis Project: www.mathedleadership.org/ccss/materials.html for tools to assist with curriculum study. Three tools were developed to provide detailed information about the extent to which curriculum materials align with and support the implementation of the CCSS. Tool 1 focuses on mathematics content trajectories, Tool 2 focuses on mathematical practices, and Tool 3 focuses on important considerations complementary to the standards like equity, assessment, and technology that can impact implementation of mathematics curricula.

To further assist your curriculum study and implementation, the *CCSS Guide to Core-Plus Mathematics* has been developed. This booklet provides page number references for each CCSS Standard and the aforementioned CCSS Pathway recommendations for Courses 1–3. This booklet is downloadable from ConnectED. In addition, the CCSS Standards that apply to each lesson for Courses 1–4 are listed in the margin of the lesson and investigation openers of the corresponding *Teacher's Guide* for the course.

Other useful tools for analyzing mathematics programs are the *High School Publishers' Criteria for the Common Core State Standards for Mathematics* (Spring 2013), the mathematical practices rubric developed by the *Institute for Advanced Study/Park City Mathematics Institute* (Summer 2011), the *Instructional Materials Analysis* from the Dana Center in Austin, Texas, and the *EQuIP rubric by Achieve*. Some of the above resources may be helpful to review whether or not you are in a CCSS adoption state. A resource helpful to all districts in states either implementing or not implementing the CCSS is *Choosing a Standards-Based Curriculum*. Selected chapters are downloadable from www2.edc.org/mcc/pubs/mguide.asp. The book can be ordered from www.heinemann.com/products/E00163.aspx.

Piloting Considerations

When considering the adoption of mathematics textbooks, districts often decide to pilot two or more programs to feed into their program review. If your district mathematics goals for student learning align with the problem-based, inquiry-oriented *Core-Plus Mathematics* program, you may wish to pilot a course or selected units to inform decisions. If so, we recommend that pilot teachers receive professional development based on the content they will be teaching. If your district is shifting to inquiry-based instruction, when evaluating the pilot, keep in mind that it takes time to change classroom expectations and norms. There may be some push-back initially from students who have been successful in mathematics courses that have required memorization and imitation rather than learning through problem-solving, reasoning by sense-making, and by discussing mathematical ideas with classmates. Some students may find mathematics learning more difficult when the expectations are shifted. Other students who have been less successful in earlier mathematics courses may find that they are learning more with the approach of the *Core-Plus Mathematics* program.

If you are piloting within your current mathematics course offerings, you might consider piloting Course 1, Unit 2, *Patterns in Data*. This unit may provide students the opportunity to learn statistics-related standards that your current course does not and make use of the affordances of the *CPMP-Tools* built-in data sets and statistical software. If you wish to pilot material from the algebra and functions strand, consider teaching the CCSS Pathway outlined in the Unit Planning Guide for Course 1, Unit 1, *Patterns of Change*, and Unit 3, *Linear Functions*. As you select pilot material, keep in mind the trade-offs, particularly within mathematical strands, of teaching a later unit without teaching earlier units in the strand. Try to retain the coherence and connectedness purposefully developed into the curriculum.

Initial Considerations

Building a Strong Foundation

Careful consideration should be given to many issues as you plan for an effective implementation of the *Core-Plus Mathematics* program. If you are a considering a pilot to inform adoption, suggestions from schools that have implemented the program may be helpful as you develop an implementation plan.

- Begin adoption with Course 1 and add a course level each year. Encourage teachers to progress from Course 1 to Course 4 in stages, so they can develop a thorough understanding of the curriculum and the growth of mathematical ideas across the courses.
- Schedule classes to allow for common planning periods for teachers teaching the same course.
- Produce a Frequently Asked Questions document containing your district's responses to community questions so that there is consistent message from district administrators, mathematics teachers, counselors, teachers of other subject areas, and school office staff.
- Consider how district and individual teacher decisions can affect the amount of material taught each year. (See Pacing Considerations, page 40, and the CCSS Pathways in the *Teacher's Guide* Unit Planning Guides.)
- Assess classroom organization needs, such as tables and chairs rather than desks for students.
- Assess district technology needs. Handheld technology (such as the TI-84) and *CPMP-Tools* should be available for each student. Some districts provide access to handheld technology for each student to use at home, as well as a set of handheld technology for each mathematics classroom. (See the *CPMP-Tools* overview on pages 14–18.)
- Consider ways to align your district curriculum with your district goals and state mathematics standards. (See Curriculum Adoption: Advice and Tools on pages 18–20.)
- Develop an ongoing professional development plan that will provide continuous support for high school, middle school, and special education teachers, and paraprofessionals. High quality professional development will weave together mathematics, pedagogy, and assessment and allow time for teachers to become familiar with new technology.
- Develop a professional development plan for newly hired teachers, such as a common planning period with mentor teachers and additional summer or academic year professional development opportunities.

- Consider providing collaborative learning, technology, literacy, and alternative assessment workshop opportunities for mathematics teachers before they begin teaching the curriculum.
- Provide opportunities for instructional leaders, particularly building principals, to understand the goals of your mathematics program and discuss ways to promote effective classroom instruction. (One professional development option for leadership teams is workshops based on the three *Lenses on Learning* modules: www2.edc.org/CDT/cdt/cdt_lol1.html.)

Acceleration

As noted previously, differentiation to challenge students is designed into the *Core-Plus Mathematics* program. But if your district has a history of accelerating some students, you may wish to continue this approach.

There is a trend in most states to include in middle school expectations some algebra and geometry content that historically has been taught in high school. The increased middle school expectations makes it more challenging to develop accelerated courses and to select students into these courses. If this is the case in your state, it may not be advisable for students to skip a middle school course in order to accelerate. One method of acceleration suggested in the *Common Core State Standards*, Appendix A is to compact middle school courses.

Some methods for acceleration to allow some students to take Advanced Placement Calculus or Advanced Placement Statistics their senior year include:

- Provide an 8th-grade course that includes middle school expectations and some of the content from *Core-Plus Mathematics* Course 1. (High expectations in this 8th-grade course is one way to determine whether a student has the ability and work ethic to maintain accelerated classes through high school. If not, the student could enroll in Course 1 in 9th grade.) Then in 9th grade, develop a course that includes the remainder of Course 1 and Course 2 content.
- Compacting courses may occur by teaching Courses 1–3, and *Course 4: Preparation for Calculus* or *Transition to College Mathematics and Statistics* in grades 9 through 11.
- Provide a summer Course 1 for 8th-grade students so they may enroll in Course 2 as 9th-graders.
- In schools with semester block scheduling at the high school level, a student may enroll in two courses in a given year.
- In schools with academic-year schedules, two mathematics classes may be scheduled back-to-back to allow a class of students to study one course in the first semester and the next course in the second semester.
- Students who have not accelerated earlier may choose as seniors to double up on classes by enrolling in both *Course 4: Preparation for Calculus* or *Transition to College Mathematics and Statistics* and AP Statistics.

A Word of Caution: Fourth-Year Mathematics Courses

Data show that students who are not enrolled in a mathematics course their senior year are much more likely to be placed in a remedial (non-credit bearing) course upon admission to college. In some schools, students may elect to take a statistics or discrete mathematics course—courses that frequently do not provide the mathematical content to be successful on college placement tests.

Based on the experiences in some schools using *Core-Plus Mathematics*, motivated students can be successful in an AP Statistics course following successful completion of Course 3. But this is not recommended. Feedback from some students going on to colleges indicated that even if students successfully gain an AP Statistics credit in high school, they may be placed into a remedial non-credit course in college. Some schools now require both student and parent signatures on a waiver indicating that they understand the risk involved with the decision to omit a fourth year of college preparatory mathematics in high school to enroll in AP Statistics. For a fourth-year high school mathematics course continuing the preparation of students for college and careers, you may wish to consider *Transition to College Mathematics and Statistics* published by McGraw-Hill Education.

Local Mathematics Program Evaluation

As noted earlier, district choices for curriculum should be influenced by local mathematics goals for student learning. Program evaluation should be designed to gather data on all of your student learning goals.

Observations by teachers and administrators will provide valuable information for you to evaluate 21st-Century skills such as critical thinking, communication, collaboration, and creativity (www.p21.org). Student attitudes, including conceptions about what "doing mathematics" means as well as perseverance, well-founded confidence, and enthusiasm for doing mathematics, can be documented by observation or, more formally, by using belief and attitude inventories. Observations can also provide valuable information on student progress toward some of the CCSS mathematical practices, such as making sense of problems and persevering in solving them, critiquing the reasoning of others, and using appropriate tools strategically. Teachers' assessments of students' progress toward these goals should be highly valued. Teachers' professional knowledge of students' progress towards these goals prior to implementation of *Core Plus Mathematics* will provide a baseline for comparison.

The Inside Mathematics project includes tools for observation and reflection that may be useful as teachers shift to classroom instruction that develops these learning goals for students. (See Additional Resources page 76.)

Data for program evaluation that are focused on mathematics expectations will come from state-mandated assessments. If these state assessments have or are undergoing change, baseline data for years prior to implementing *Core-Plus Mathematics* may not be available.

While state testing results are one important source for program evaluation, trend data are often helpful for evaluating continuous improvement in mathematics programs. Some districts using *Core-Plus Mathematics* have tracked trend data for high school students by using pre- and post-implementation years of ACT and SAT tests results. If the demographics of your district are stable and the percentage of students taking these college readiness assessments has not changed over time, the ACT or SAT results can be used as one indicator of progress toward your mathematics program goals. Trends in enrollment patterns and achievement results for Advanced Placement Mathematics and Statistics exams are another measure that some districts have used to evaluate their mathematics programs. (See pages 71–72.) Other measures used by districts implementing *Core-Plus Mathematics* have been failure rates, performance in science courses and on science sections of standardized tests, mathematics competitions, scholarships and awards such as the National Merit Scholars, college placement test results, and surveys of high school graduates.

The Mathematics Assessment Project (MAP; www.mathshell.org/ba_mars.htm) is developing formative assessment lessons and rich summative performance tasks to support the *Common Core State Standards* emphasizing the vital mathematical practices they require. These tasks might be used as formative and summative evaluation of your mathematics program.

Overall, your program evaluation plan should be developed to align with your program goals.

Communicating with Parents

Parents are understandably concerned about the implementation of any major curriculum or instructional changes in programs that affect their children. Their understanding and support will play an important role in the successful implementation of the *Core-Plus Mathematics* program. This understanding and support can also be a major factor in their children's success with mathematics. For many parents (who attended high school in the United States), the sequence of Algebra, Geometry, Advanced Algebra, and Precalculus courses was the gateway to college. The *Core-Plus Mathematics* curriculum provides an alternative international-like, integrated mathematics route to both college and careers in the technology-based workplace of the 21st century. Its organization is similar to that of high-performing countries on international assessments.

It is important that parents are provided information about *Core-Plus Mathematics* well in advance of its implementation and periodically while it is in use. Schools participating in the testing and evaluation of the curriculum organized “Math Nights” where parents were provided a rationale for the proposed changes in curriculum and instructional practices, including expectations of colleges, small businesses, and industry; were guided through the mathematics of the curriculum; and were given opportunities to raise questions or concerns. Opportunities for parents to review the textbooks permit them to see that the texts include important ideas and methods of algebra, geometry, and functions—and much more: statistics, probability, trigonometry, and discrete mathematics. Parents also see in the texts the kinds of mathematics that they might possibly use in their jobs. Parents can be referred to the Parent Resource at www.wmich.edu/cpmp/parentresource.html for assistance in understanding the curriculum and in helping their children. Follow-up “Math Nights” can feature upcoming content, technology assistance, and student presentations.

As noted in the next section on college admissions, students studying *Core-Plus Mathematics* have been admitted to highly selective national colleges and universities. This information and information from your local colleges should be made available to parents. It is also important to note that increasing numbers of colleges and universities are offering calculus programs that share many of the features of this curriculum: emphasis on developing conceptual understanding and presenting problems in context, use of technology tools such as graphing calculators or computers, collaborative group work, student projects, and writing in and about mathematics.

Sample Parent Letter: Units 1–2

As an initial step toward communication, you might consider sending periodic information letters to parents. A sample letter for Course 1 Units 1 and 2 follows on page 26.

Throughout the school year, you may wish to periodically send additional letters to parents. Pages 27–28 include background information for Course 1 Units 3–8 and other ideas for material to include in parent letters.

[school letterhead]

[date]

Dear Parent or Guardian:

We would like to share with you information about our new mathematics program. The program was developed by the Core-Plus Mathematics Project (CPMP) through a grant from the National Science Foundation. The text, *Core-Plus Mathematics*, is based on national standards for curriculum and teaching developed by the National Council of Teachers of Mathematics and endorsed by 15 mathematical sciences organizations. The program also aligns with the Common Core State Standards for Mathematics. The text also reflects the needs of business and industry today, who are calling for 21st century workers who can think and reason about quantitative situations, who are innovative, who can communicate effectively, and who can work together in teams. In each course, students study algebra, geometry, statistics, and discrete mathematics.

The first two units of Course 1 are devoted to the study of important and broadly useful topics in algebra and statistics.

> In Unit 1, *Patterns of Change*, students extend their understanding and skill in algebra in three ways. They learn how to recognize relationships among independent and dependent variables in problems and experiments and to describe patterns in quantitative variables that change over time. They learn how to read and construct data tables and graphs that display relationships among variables. They continue developing symbol sense—the ability to connect important patterns of change to linear, exponential, quadratic, and inverse variation rules. After completing this unit, your student should be able to solve problems like 1–4 on pages 70–72 of the text.
>
> In Unit 2, Patterns in Data, students learn to organize and analyze data using various graphical displays (histogram, dot plot, box plot, and stem-and-Ieaf plot) and to summarize data using measures of center (mean, median, mode) and measures of variability (range, interquartile range, percentiles, and standard deviation). For the kinds of problems your student should be able to solve after completing the unit, see Tasks 1–5 on pages 145 and 146 of the textbook.

Your student should be developing a Math Toolkit, which summarizes concepts, facts, skills, and methods that he or she is learning. You may wish to review his or her Math Toolkit from time to time or consult it as you are providing assistance on homework.

Through our implementation of the *Core-Plus Mathematics* program, your student will develop a new excitement about mathematics and will grow confident in his or her ability to think mathematically. You can contribute to this growth by supporting completion of work that is assigned as homework. A parent resource is available at: www.wmich.edu/cpmp/parentresource.html

If you have any questions about our mathematics program, please do not hesitate to contact us.

Sincerely yours,
[Local principal, mathematics department chair, teacher]

Units 3–8 Content Overviews

In Unit 3, *Linear Functions*, students learn how to recognize situations in which key variables change at a constant rate. They learn how to express and interpret those patterns of change in data tables, slopes and intercepts of straight-line graphs, and equations in the form $y = a + bx$. They learn techniques for solving linear equations and inequalities that arise in science and business problems. For the kinds of problems your student should be able to solve after completing this unit, see Tasks 1–7 on pages 232–236 of the textbook.

In Unit 4, *Discrete Mathematical Modeling*, students learn basic concepts of graph theory. They use vertex-edge graphs to model and solve problems related to many different types of networks, including communication, computer, transportation, and distribution networks. Problems involve finding the optimal (best) route and avoiding conflict among objects. Upon completion of this unit, your student should be able to solve problems like 1, 2, and 3 on pages 286–288.

In Unit 5, *Exponential Functions*, students learn how to construct and use data tables, graphs, and equations in the form $y = a(b^x)$ to describe and solve problems about exponential relationships such as population growth, investment of money, and decay of medicines and radioactive materials. Upon completing this unit, your student should be able to solve problems like 3–7 on pages 357 and 358.

In Unit 6, *Patterns in Shape,* students learn to describe, classify, and visualize two-dimensional and three-dimensional shapes. They consider the Pythagorean Theorem and triangle congruence relationships. They also use experimentation and reasoning to investigate properties of polygons and polyhedra. For the kinds of problems your student should be able to solve after completing this unit, see Tasks 3, 4, 7, 8, and 9 on pages 457–459.

In Unit 7, *Quadratic Functions*, students learn to identify quadratic patterns in tables, graphs, and problem conditions. They write quadratic functions to represent situations, combine terms and factor expressions, and solve quadratic equations. The concepts and skills developed in this unit will be practiced in Unit 8 and units in later courses. Upon completing this unit, your student should be able to solve problems such as 3–6 on pages 528 and 529.

In Unit 8, *Patterns in Chance*, students learn to construct probability distributions and use the Addition Rule to solve problems involving chance. They design and carry out simulations to estimate answers to questions about probability. See Tasks 1–5 on pages 586–588 for samples of the kinds of problems your student should be able to solve after completing this unit.

Other Possible Topics for Parent Letters

One important topic to address is how parents can help with homework. In addition to providing the typical advice about providing a quiet location, technology access at home (if possible), and holding expectations that homework will be completed, you could provide some or all of the questions on the next page to help parents guide their children. Another possible topic is preparation for standardized tests.

General Questions to Ask Your Student—Sample Text

To support learning, it is best to not do homework problems for your student, even if this is accompanied by an explanation. Instead, we suggest you ask questions like the following. Questions such as these work to enhance learning and success, whether the homework question is very basic, or very complex. This is a good place to start all help sessions. (Spanish versions of these questions are available from the CPMP Parent Resource Web site or your student's teacher.)

- What have you been doing in class that relates to this problem? Can you explain the main ideas to me so I can think about the problem with you?
- Do you have some examples in your notes or toolkit that would help us think about this problem? Did your class do a Summarize the Mathematics recently that would be relevant?
- What do you know right now and why is that not enough to do the problem?
- Explain these vocabulary words to me. Are there other words that you don't understand?
- What have you tried? Explain the steps to me. Can you explain this another way?
- Is there a way to organize this, with a sketch or a diagram or a graph or a table, that might help us get a handle on the problem?
- If you cannot complete the problem, can you make a simpler problem that you can complete?
- What question will you ask your teacher tomorrow?
- Is there someone else on your team that you can call to discuss this task? Before calling, think about what you wish to discuss.
- Now that you have a solution, does it make sense? Can you check your solution? Have you answered clearly and completely? Convince me.

Practicing for Standardized Tests—Sample Text

In addition to the practice embedded in problems that students do in class and in Applications, Connections, Reflections, and Review homework tasks, we periodically provide practice in the format used by many standardized tests, such as state tests and the ACT and SAT tests. Attached to this letter is one such practice set. Although some of the items may look unfamiliar to students, they should have the background knowledge and problem-solving abilities to complete these items. (Attach a Practicing for Standardized Tests set. See Course 1 *Unit Resource Masters* pages 73 and 74.)

College Admissions

Since most college preparatory curricula in the United States historically have been sequences of Algebra 1, Geometry, Advanced Algebra, and Precalculus, implementation of *Core-Plus Mathematics* may raise questions related to college or university admissions for students who have successfully completed three or four years of this program.

Some people are not aware that integrated high school mathematics programs are the norm in countries other than the United States. College admissions departments accepting students from overseas commonly receive transcripts indicating students have studied mathematics courses in high school without any designation of courses such as a Geometry or Advanced Algebra. (Community college admissions departments may see fewer applications from outside of the U.S.)

In addition, integrated high school mathematics courses such as *Core-Plus Mathematics* and other NSF-funded high school curricula have been published since the late 1990s. Many higher education admissions offices are aware of these programs. Graduates who have studied *Core-Plus Mathematics* have been accepted into a wide variety of higher education institutions. The first graduates from *Core-Plus Mathematics* began enrolling in colleges in the year 2000. Schools have *not* seen any students refused admittance to a college or university due to their study of *Core-Plus Mathematics*. A comprehensive list of these colleges is far too long to include in this resource, but a list of selective national colleges where students have been admitted prior to 2012 is included on page 30.

Titles of Courses

In cases where university mathematics departments have reviewed course descriptions of *Core-Plus Mathematics*, these courses have been approved as meeting admissions requirements of the institutions. Courses 1–3 are considered the equivalent of the college preparatory Algebra-Geometry-Advanced Algebra sequence. High school counselors can submit copies of the course descriptions found on pages 8–11 of this guide or summary paragraphs based on those course descriptions. Such course descriptions with accompanying transcripts showing Integrated Mathematics I, II, III, and IV or Mathematics 1, 2, 3, and 4 have proved sufficient for admissions purposes. Some schools have used course titles such as Integrated Algebra/Geometry or Integrated Algebra/Geometry/Statistics. The textbook title, *Core-Plus Mathematics*, should not be used as a course title.

NCAA Eligibility

For student athlete eligibility, the NCAA regularly examines courses based on short course descriptions of the mathematical content rather than textbook titles or course titles. Schools have included the unit descriptions on pages 8–11 of this book when submitting courses to NCAA for review. Submitting unit descriptions has ensured the courses derived from *Core-Plus Mathematics* meet NCAA requirements.

Selective National Colleges and Universities

Sometimes the question arises about whether students who have studied *Core-Plus Mathematics* have gained admission into exclusive or selective universities. Teachers of *Core-Plus Mathematics* have reported that their students have been accepted into 32 of the 35 top-ranked universities as ranked by *U.S. News and World Report* in 2012.

Selective National Universities That Have Accepted CPMP Students

Boston College
Brandeis University
Brown University
Carnegie Mellon University
Columbia University
Cornell University
Dartmouth College
Duke University
Emory University
Georgetown University
Harvard University
Massachusetts Institute of Technology
New York University
Northwestern University
Princeton University
Rice University
Stanford University
Tufts University
University of California–Berkeley
University of California–Los Angeles
University of Chicago
University of Michigan–Ann Arbor
Univers1ty of North Carolina–Chapel Hill
University of Notre Dame
University of Pennsylvania
University of Rochester
University of Southern California
University of Virginia
Vanderbilt University
Wake Forest University
Washington University in St. Louis
Yale University

As of the year 2012, the developers of *Core-Plus Mathematics* have not received information from schools indicating that any graduates have applied to the other three top 35 national universities as ranked by *U.S. News and World Report*:

California Institute of Technology
Johns Hopkins University
College of William and Mary

The manner in which students encounter mathematical ideas can contribute significantly to the quality of their learning and the depth of their understanding. *Core-Plus Mathematics* is designed so that students engage in the mathematical behaviors and habits of mind identified in the CCSS Standards for Mathematical Practices as the primary vehicle for learning mathematics and statistics. This is evident in the nature of the student materials themselves and in the instructional model described below.

The *Core-Plus Mathematics* program supports teachers who wish to develop 21st-century learning and innovation skills (such as critical thinking and problem solving; communication and collaboration; and creativity and innovation) as recommended by the Partnership for 21st Century Skills (www.p21.org). The P21 mission is "to serve as a catalyst to position 21st-century readiness at the center of US K12 education by building collaborative partnerships among education, business, community and government leaders."

This section outlines the *Core-Plus Mathematics* instructional model and provides specific ideas to assist in developing creativity and maintaining classroom environments for the 21st century.

Instructional Model

Each lesson includes a cluster of 2–4 focused and connected mathematical investigations that engage students in a four-phase cycle of classroom activities, described in the following paragraphs—*Launch, Explore, Share and Summarize*, and *Self-Assessment*. This cycle is designed to engage students in investigating and making sense of problem situations, in constructing important mathematical concepts and methods, in generalizing and proving mathematical relationships, and in communicating, both orally and in writing, their thinking and the results of their efforts. Most classroom activities are designed to be completed by students working collaboratively in groups of two to four students punctuated by whole-class discussions (that retain the cognitive demand of the problems) once students have had time to discuss the activities and have some partial understanding of the mathematical ideas being developed.

LAUNCH class discussion

Think About This Situation

The lesson launch promotes a teacher-led discussion of a problem situation and of related questions to think about. This discussion initiates the context for the student work to follow and helps to generate student interest. It also provides an opportunity for the teacher to assess student knowledge, to discuss cultural and mathematical context suppositions, and to clarify directions for the investigation to follow. The discussion sets the stage for students to make sense of and persevere in solving the investigation problems. The discussion should not be lengthy nor pre-teach the lesson.

Mary Walz Prairie du Sac, WI, Sauk Prairie High School.

EXPLORE group investigation

INSTRUCTIONAL NOTE

You may wish to have a quick whole class discussion around Problems 3–5 before most groups begin Problem 6.

Investigation

Classroom activity then shifts to investigating focused problems and questions related to the launching situation by gathering data, looking for and explaining patterns, constructing models and meanings, and making and verifying conjectures. As students collaborate in pairs or small groups, the teacher circulates among students providing guidance and support, clarifying or asking questions such as why students think their answer is reasonable, giving hints, providing encouragement, and drawing group members into the discussion to help groups collaborate more effectively. Periodically, it may be valuable to facilitate a whole class discussion particularly at the close of the class period. The investigations and related questions posed by students and teachers drive the learning and offer many opportunities to engage in the CCSS mathematical practices.

SHARE AND SUMMARIZE class discussion

Summarize the Mathematics

This investigative work is followed by a teacher-led class discussion in which students summarize and explain the reasoning supporting mathematical ideas developed in their groups, providing an opportunity to construct a shared understanding of important concepts, methods, and justifications. This discussion leads to a class summary of important ideas or to further exploration of a topic if competing perspectives remain. Varying points of view and differing conclusions that can be justified should be encouraged. This discussion based on student thinking during the investigation is crucial to building understanding of mathematical ideas for the procedural skill development to follow.

Mathematics Toolkits

Students learn mathematics by doing mathematics. However, it is important that students prepare and maintain summaries of important concepts and methods that are developed. Students should create a Mathematics Toolkit that organizes important class-generated ideas and selected Summarize the Mathematics responses as they complete investigations. Prompts for the Mathematics Toolkit are provided in the *Teacher's Guide* to assist in identifying and summarizing key concepts and methods as they are developed by students.

Looking Back Lesson

The final lesson in each unit helps students review and synthesize the key mathematical concepts and techniques developed in the unit. The tasks in this lesson help students pull together and demonstrate what they have learned in the unit and, at the same time, provide helpful review and confidence-building for students. The Summarize the Mathematics questions in the Looking Back lesson are focused on key ideas of the unit, and the Check Your Understanding asks students to prepare a summary of the important concepts and methods developed in the unit. Templates to guide preparation of these unit summaries may be found in the *Unit Resource Masters*. Completed unit summaries should become part of students' Mathematics Toolkits.

Berett Wilbur

SELF-ASSESSMENT individual tasks

James Poulson/CPMP

Check Your Understanding

Students are given an individual task to complete to check and reinforce their initial understanding of concepts and procedures. Teachers should encourage students to develop the ability to determine for themselves whether their understanding is enough to secure progress to the tasks found in the On Your Own homework sets.

On Your Own Homework Sets

In addition to the classroom investigations, *Core-Plus Mathematics* provides sets of On Your Own homework tasks, which are designed to engage students in applying, connecting, reflecting, extending, and reviewing their evolving mathematical knowledge. On Your Own tasks are provided for each lesson in the materials and are central to the learning goals of each lesson. The chart below describes the types of tasks in On Your Own (OYO) sets.

On Your Own: Homework Tasks	
Applications	These tasks provide opportunities for students to use and strengthen their understanding of the ideas they have learned in the lesson.
Connections	These tasks help students to build links between mathematical and statistical topics they have studied in the lesson and to connect those topics with other mathematics that they know.
Reflections	These tasks provide opportunities for students to re-examine their thinking about ideas in the lesson.
Extensions	These tasks provide opportunities for students to explore further or more deeply the mathematics they are learning.
Just-In-Time Review and Distributed Practice	These tasks provide opportunities for just-in-time review of concepts and skills needed in the following lesson and distributed practice of mathematical skills to maintain procedural fluency. A clock icon near the solution in the *Teacher's Guide* designates just-in-time review tasks.

These tasks in the OYO are intended primarily for individual work outside of class. If a few students are identified as needing additional assistance with specific skills, they should be given additional assistance outside of class. Selection of homework tasks should be based on student performance, the availability of time, and technology access. Also, students should exercise *some choice* of tasks to pursue, and at times should be given the opportunity to pose their own problems and questions to investigate.

Practicing for Standardized Tests

The online eAssessments for *Core-Plus Mathematics* include sample items similar to the released items from the *Common Core State Standards for Mathematics* assessment consortia as well as sample SAT and ACT items. These items can be used periodically as a classroom "warm-up" item or used with selected students who need additional practice with these types of multiple-choice tasks.

Opportunities for additional review and practice are also provided in the Practicing for Standardized Tests masters in the *Unit Resource Masters*. Each Practicing for Standardized Tests master presents ten questions that draw on all content strands. The questions are presented in the form of test items similar to how they often appear in standardized tests such as state assessment tests, the Preliminary Scholastic Aptitude Test (PSAT), or the ACT PLAN. We suggest using these practice sets following the unit assessment so students can become familiar with the formats of standardized tests and develop effective test-taking strategies for performing well on such tests.

Planning for Instruction

The *Core-Plus Mathematics* curriculum is not only changing what mathematics all students have the opportunity to learn, but also changing how that learning occurs and is assessed. Active learning is most effective when accompanied with active teaching. Just as the student texts are designed to actively engage students in doing mathematics, the teacher resource materials are designed to support teachers in planning for instruction; in observing, listening, questioning, and facilitating student work; in orchestrating classroom discussion; and in managing the classroom.

In the *Teacher's Guide*, you will find teaching notes for each lesson, including instructional suggestions and sample student responses to investigation problems and On Your Own tasks. Thinking about the range of possible responses and solutions to problems in a lesson proves to be very helpful in facilitating student work. Each of the features listed below is included in the teacher materials for each unit:

- Unit overview and lesson overviews
- A Planning Guide including suggested pacing, homework assignments, and resources
- Facing student text with corresponding teacher material, including solutions for investigation problems and On Your Own tasks
- Detailed instructional notes and other suggestions for each phase of the instructional model
- Promoting Mathematical Discourse scenarios

Planning Guide

At the beginning of the teacher materials for each unit is a Planning Guide that provides information about materials needed, time guidelines, and recommended assignments from the On Your Own sets.

The Unit 1 Planning Guide for Course 1 is shown below. You will notice in the Planning Guide that the suggested time for the second lesson is approximately five 50-minute instructional periods. This may vary depending on the background and make up of the class. Remember: *Developing deep understanding is more important than just "completing activities."*

Since acquiring the physical materials requires advance planning, specific materials for each lesson are listed under Resources. Although it is not stated in the Planning Guide, it is assumed that students have access to technology at all times for in-class work.

The developers recommend that the homework assignment from the On Your Own sets not be held off until the end of the lesson or the investigation just preceding the On Your Own set. Some teachers choose to post On Your Own assignment at the beginning of a lesson along with the due date, usually a day or two following the planned completion of the lesson. Other teachers prefer to assign selected On Your Own tasks at appropriate times during the course of the multiday lesson and then assign the remaining tasks toward the end of the lesson. Note that all recommended assignments include provision for student choice of some tasks. This is but one of many ways in which this curriculum is designed to accommodate and support differences in students' interests and performance levels. Additionally, it is strongly recommended that student solutions to Connections tasks be discussed in class, providing students with an opportunity to compare and discuss student work and synthesize key ideas within the classroom.

UNIT 1 Planning Guide

Lesson Objectives	Pathways: Pacing and OYO Assignments*	Resources
Lesson 1 Cause and Effect • Develop disposition to look for cause-and effect relationships between variables • Review and develop skills in organizing data in tables and graphs and using words to describe patterns of change shown in those representations • Review or begin to develop knowledge about common patterns of change (linear, inverse, exponential, quadratic) and ability to use symbolic rules to represent and reason about those patterns • Use tables, graphs, and rules to solve problems of cause-and-effect change	**CCSS Pathway** Optional, depending on students' middle school background **CPMP Pathway** Optional, depending on students' middle school background	For each group of students: • Rubber bands and fishing weights, bags of nuts and bolts, or other weights • Meter sticks • Dice • Three different coins • Unit Resource Masters
Lesson 2 Change Over Time • Develop ability to recognize recursive patterns of change • Develop ability to use calculators to iterate stages in ...ursive pattern	**CCSS Pathway** (5 days, includes assessment) **Investigation 1:** OYO—choose one of A1–A4, A5 or A6, C10, C14, R19, E22 or E23, Rv26–Rv29	• Access to computers with spreadsheet software, *CPMP-Tools*, or calculators with spreadsheet

Orchestrating Lessons

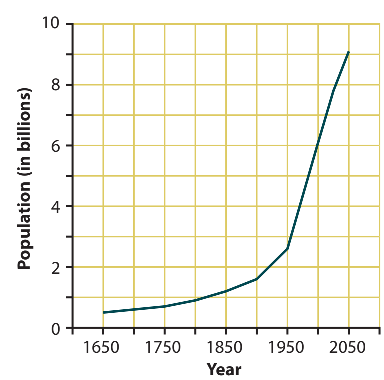
The *Core-Plus Mathematics* program is designed to engage students actively in a four-phase cycle of classroom activities (See Instructional Model on pages 31–34 of this guide.) The activities often require both students and teachers to assume roles quite different than those in more traditional mathematics classrooms. Becoming accustomed to these new roles usually takes time, but teachers report that the time and effort required are worthwhile in terms of both student learning and professional fulfillment. Although realistic problem solving and investigative work by students is the heart of the curriculum, how teachers orchestrate the launching of an investigation and the sharing and summarizing of results is critical to successful implementation. Lessons should be introduced by asking students to think about a situation such as the one reproduced below, which is used to launch Lesson 2 of the *Patterns of Change* unit in Course 1.

Change Over Time

Every 10 years, the U.S. Census Bureau counts every American citizen and permanent resident. The 2010 census reported the U.S. population to be 308.7 million, with growth at a rate of about 0.8% each year. The world population in 2011 was over 7 billion and growing at a rate that will cause it to exceed 10 billion by the year 2050.

National, state, and local governments and international agencies provide many services to people across our country and around the world. To match resources to needs, it is important to have accurate population counts more often than once every 10 years. However, complete and accurate census counts are very expensive.

World Population 1650–2050



Source: www.census.gov/ipc/www/world.html

THINK ABOUT THIS SITUATION

The population of the world and of individual countries, states, and cities changes over time.

a How would you describe the pattern of change in world population from 1650 to 2050?

b What do you think are some of the major factors that influence population change of a city, a region, or a country?

c How could governments estimate year-to-year population changes without making a complete census?

Students enter the classroom with different backgrounds, experience, and knowledge. These differences can be viewed as assets. Engaging the class in a free-flowing, give-and-take discussion of how students think about the launch situations serves to connect lessons with the informal understandings of data, shape, change, and chance that students bring to the classroom. The teacher should try to maximize the participation of students in these discussions by emphasizing that their ideas and possible approaches are valued and important and that definitive answers are not necessarily expected for all launch questions.

Once launched, a lesson may involve students working together collaboratively in small groups for a period of days, punctuated occasionally by brief, whole-class discussions of questions students have raised. In this setting, the lesson becomes driven primarily by the instructional materials themselves. Rather than orchestrating class discussion, the teacher shifts to circulating among the groups and observing, listening, and interacting with students by asking guiding or probing questions. These small-group investigations lead to (re)invention of important mathematics that makes sense to students. Sharing and agreeing as a class on the mathematical ideas that groups are developing is the purpose of the Summarize the Mathematics in the instructional materials.

As a general rule, students first should decide as a group on responses to Summarize the Mathematics questions. The sample Summarize the Mathematics shown below is the first of two in Lesson 2 of the *Patterns of Change* unit in Course 1.

SUMMARIZE THE MATHEMATICS

In the studies of human and whale populations, you made estimates for several years based on growth trends from the past.

a What trend data and calculations were required to make these estimates:

- **i.** The change in the population of Brazil from one year to the next? The new total population of that country?
- **ii.** The change in number of Alaskan bowhead whales from one year to the next? The new total whale population?

b What does a *NOW-NEXT* rule like $NEXT = 1.03 \cdot NOW - 100$ tell about patterns of change in a variable over time?

c What calculator commands can be used to make population predictions for many years in the future? How do those commands implement *NOW-NEXT* rules?

Be prepared to share your thinking with the class.

Summarize the Mathematics class discussions are orchestrated somewhat differently than the launch of a lesson. At this stage, mathematical ideas and methods still may be under development and may vary for individual groups. So, class discussion should involve groups comparing their methods and results, analyzing their work, and arriving at conclusions agreed upon by the class. Potentially productive class discussions allow for how a given solution differs from another, and why students used particular methods, rather than merely explaining how they solved a task.

Sample discourse scenarios are available in the *Teacher's Guide*. These sample discussions, titled Promoting Mathematical Discourse, offer possible teacher-student discourse around selected Summarize the Mathematics and Think About This Situation questions. Teachers may choose to review the Promoting Mathematical Discourses with a colleague and record additional questions they might choose to ask during the class discussion. Following the class discussion, some teachers make notes on a copy of the discourse scenario to inform preparation for the following year.

PROMOTING MATHEMATICAL DISCOURSE

UNIT 1

Summarize the Mathematics, *page 31*

Teacher: Let's summarize our thinking from Investigation 1. You made estimates of human and whale populations based on growth trends from the past. Take a look at Part ai. What trend data and calculations did you use to predict the population change in Brazil from one year to the next? And also the new total population?

Carley: There were births of 1.5% and deaths of 0.6% each year. You took the population and multiplied by 0.009 to get the next answer.

Teacher: What do the rest of you think?

Tony: Where did the 0.009 come from again? 1.5% − 0.6% is 0.9%. Oh, 0.9% is the same as 0.009. So, the next population is 0.009 times the previous population.

Teacher: What does the 0.9% represent?

Tony: That is the final growth rate made up of the birth and death rates.

Teacher: So, look again at Part a. What did we just answer?

Caitlyn: We were talking about the population change and not the new total population.

Ken: Yeah, you need to add in the current population to get the new total.

Teacher: What information were we using to get these results?

Chloe: We used the birth rate, death rate, and the current

Teacher: But it seems that there may be an error to me. The first rule for change uses *NOW* times 0.009 and the second one uses 0.009 times *NOW*. Isn't that a problem?

Carley: Yes, we should have the same thing in both rules.

Thomas: No, it is okay this way. It is like 3 times 4 is the same as 4 times 3. It doesn't matter which order two numbers are multiplied.

Kyle: Well that is true, but I would still like to have them written the same way. Either way is fine, but both the same way.

Teacher: Okay, are we agreed that these rules are correct as written? *(Students agree.)*

Then you may choose the way that you would like to represent the yearly population change and the total population for your summary. As you finalize your answer to i of Part a, complete ii related to the whale population growth that you investigated. Since you used a variety of growth rates and hunting quotas for the whale population, look for a way to answer the questions in a more general form than we did for the Brazil population question.

(Students have two minutes to work.)

Teacher: Okay class. What trend data and calculations did you use to estimate the change in number of Alaskan bowhead whales each year and the new total population changes?

Leroy: We said that you multiplied 3% times the population of 7,700, then added that to 7,700, then subtracted 50. Sometimes the numbers were different for different problems though.

Check Your Understanding tasks immediately follow the Summarize the Mathematics. As you plan, consider whether this task should be done in class or as a homework task. If you judge that students will need your assistance or peer assistance, you might ask them to begin the individual task during the class period. Similar to what is sometimes done in Japanese classes, you might then provide hints as needed for students to continue on the task. (See the TIMSS 1999 Video Study.)

CHECK YOUR UNDERSTANDING

The 2010 United States Census reported a national population of 308.7 million, with a birth rate of 1.3%, a death rate of 0.8%, and net migration of about 0.9 million people per year.

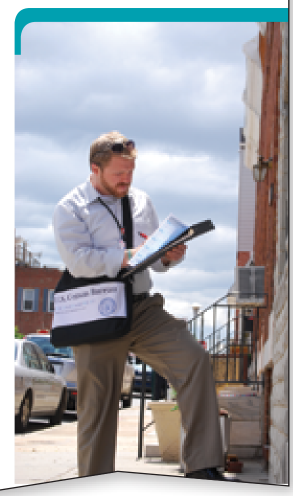
a. Use the given data to estimate the U.S. population for years 2011, 2015, 2020, 2025, and 2030.

b. Use the words *NOW* and *NEXT* to write a rule that shows how to use the U.S. population in one year to estimate the population in the next year.

c. Write calculator commands that automate calculations required by your rule in Part b to get the U.S. population estimates.

d. Modify the rule in Part b and the calculator procedure in Part c to estimate U.S. population for 2015 in case:

- **i.** The net migration rate increased to 1.5 million per year.
- **ii.** The net migration rate changed to −1.0 million people per year. That is, if the number of emigrants (people leaving the country) exceeded the number of immigrants (people entering the country) by 1 million per year.



For each part of the Summarize the Mathematics, different groups should be asked to share their responses and thinking before proceeding to the next part. Facilitate resolution of any differences as a step toward building agreement concerning the class's mathematical discoveries.

The investigations deepen students' understanding of mathematical ideas and extend their mathematical language in contexts. Technical terminology and symbolism are introduced as needed in the investigation. This sometimes occurs in the Summarize the Mathematics discussions. These discussions provide the agreement needed to inform student notes (called Mathematics Toolkits).

U.S. Census Bureau, Public Information Office (PIO)

Instructional Decisions

Pacing Considerations

Teachers at many schools that have adopted *Core-Plus Mathematics* have indicated that they recognize that it is important to allow students time to think about, explain their reasoning on, and write mathematics. Students need to have time to develop deep understandings of important mathematics. Professional judgment is called upon daily to decide how much time to allow for student work and when to insert whole-class discussions and how much time to allow for these discussions.

Many decisions made by teachers each day also affect the amount of material that students can complete in a year. Teachers who have been implementing the *Core-Plus Mathematics* program have found some successful strategies for addressing pacing challenges:

- Know the objectives and investigation focus question(s) so that you can attend to student mathematical thinking on concepts and methods.
- Students need not write complete answers to every problem in an investigation. Solutions may be considered as records of current thinking for discussion. Complete write-ups should be made at the Summarize the Mathematics in students' Mathematics Toolkits, and for homework tasks.
- When assigning an investigation, give time limits, for example, "You have 12 minutes to do Problems 1–5" or "Two more minutes until the Summarize the Mathematics discussion."
- Selectively facilitate whole-class mini-summaries before the main Summarize the Mathematics to consolidate the learning and allow students to move efficiently through the remainder of the investigation. (This may also help bring a lagging group up to speed.)
- Occasionally, an investigation problem can be assigned to individual students as homework. This problem should not be one that is too difficult. Students should be able to at least start on the problem. They should expect that they will have an opportunity to discuss their progress with others during the next class session. (This may also be a way to help bring a lagging group up to speed.)
- Check Your Understanding tasks could be assigned in pairs or as homework.
- Resist the temptation to go over all the assigned OYO tasks in class. Reserve class time for the important Connections and Reflections tasks.
- The curriculum builds on mathematics developed each year. Important topics will be practiced in review sets and revisited as students progress through the curriculum. Therefore, it is reasonable to expect mastery from different students at different times.

Teaching each of the four courses in *Core-Plus Mathematics* will help you better understand the development of mathematical concepts and methods, student retention of mathematical ideas across courses, and students' deepening understanding of mathematics. This, in turn, will give you the confidence to make daily specific teaching decisions that affect pacing.

Active Learning and Collaborative Work

The *Core-Plus Mathematics* curriculum materials are designed to promote active, collaborative learning. Creating a classroom atmosphere conducive to effective collaborative learning requires the following: an understanding of the basic philosophy behind collaborative learning, familiarity with the factors in the composition and selection of groups, and practice in classroom techniques for managing collaborative learning. The following subsections are a summary of, and a ready reference to, these aspects of the collaborative learning model. If you choose to skim through some of this material, you may wish to read the conclusion on page 51.

Philosophy Behind Collaborative Learning

Core-Plus Mathematics deliberately incorporates collaborative learning for many reasons. A collaborative environment fosters students' ability to make sense of mathematics, to reason mathematically, and to develop deep mathematical understandings. Collaborative learning is an effective method for engaging all the students in the learning process, particularly students who have been under-represented in mathematics classes. In addition, practice in collaborative learning in the classroom is practice for real life; students develop and exercise many of the same skills in the classroom that they need in their lives at home, in the community, and in the workplace. (See Partnership for 21st Century Skills: www.p21.org)

Value of Individuals Perhaps the most fundamental belief underlying the use of collaborative learning is that every student is viewed as a valuable resource and contributor. In other words, every student participates in group work and is given the opportunity and time to voice ideas and opinions. Implementing this concept is not easy. It does not happen automatically. In order to set a tone that will promote respect for individuals and their contributions, classroom norms should be established and agreed upon by the learning community. Students should be included in the process of formulating these rules. Helpful norms might include the following:

- Refer to people by the names they prefer.
- Speak respectfully by challenging ideas, not people; be open to receive constructive criticism.
- Acknowledge that everyone's ideas have value.
- Even if you disagree with others, listen to explore topics, be skeptical of ideas, and provide constructive criticism.
- Be willing to change your mind on the basis of mathematical or statistical reasoning.
- Be inclusive. Do not leave anyone out on purpose or by chance. If you are placed in groups with others whom you do not like, do not exclude them from the group discussion or from making decisions.
- Collaborate rather than compete. Group work is designed for cooperation, and one group or individual is not in competition for answers or for time with others in the classroom. As a last resort, agree to disagree.

Beth Ritsema/CPMP

You should initiate a discussion of group norms and then post them in the classroom. You should also model all of the norms correctly to show that "we" begins with "me." Those who do not adhere to the norms must accept the consequences in accordance with classroom or school disciplinary procedures.

Importance of Social Connections Even in classrooms where the rules for showing respect have been clearly established, experience has shown that students still cannot talk with one another about mathematics (or social studies, or literature, or any other subject) if they do not first have positive social connections.

One way to develop this kind of common base is through team-building activities. These short activities may be used at the beginning of the year to help students get acquainted with the whole class and may be used during the year whenever new groups are formed to help groupmates know one another better.

In one such activity, called "Whip," the members of each group give quick answers to a statement given by the teacher, such as:

- "My favorite pastime is ..."
- "My favorite vacation memory is ..."
- "Something new and positive in my life is ..."
- "A hobby or sport I like is ..."

These kinds of quick activities help students learn new and positive things about classmates with whom they may have attended classes for years but have not known well. The time taken for these quick team builders pays off later in helping students feel comfortable enough to work with the members of their group. Additional resources on collaborative learning are cited on page 76.

Need for Teaching Social Skills Experience has also shown that social skills are critical to the successful functioning of any small group. Because there is no guarantee that students of any particular age will have the social skills necessary for effective group work, it is often necessary to teach these skills in order to build a collaborative learning environment.

These social skills are specific skills, not general goals. Examples of specific social skills that you can teach in the classroom include responding to ideas respectfully, keeping track of time, disagreeing in an agreeable way, involving everyone, and following directions. Though goals such as cooperating and listening are important, general statements such as "remember to listen to your groupmates" is typically not a specific method for students to improve their social skills.

One method of teaching social skills is to begin by selecting a specific skill and then having the class brainstorm to develop a script for practicing that skill. Next, the students practice that skill during their group work. Finally, in what is called the processing, the students discuss within their groups how well they performed the assigned social skill. Effective teaching of social skills requires practicing and processing; merely describing a specific social skill is not enough. Actual practice and processing are necessary for students to learn the skill and to increase the use of appropriate behaviors during group work and at other times during class. The *Teacher's Guide* includes suggestions for specific collaboration skills to practice. Look for prompts such as those at the left.

COLLABORATION SKILL

Help group check thinking or solutions.

PROCESSING PROMPT

We checked our thinking by ... .

Composition, Selection, and Management of Groups

One of the premises of collaborative learning is that by developing the appropriate skills through practice, anyone in the class can learn to work in a group with anyone else. Learning to work in groups is a continuous process, however, and the process can be helped by decisions that you make with regard to the size, composition, method of selection, student reaction to, and duration of groups. Attention to dealing effectively with student absences also is important.

Group Size Any group must have at least two students and the largest manageable groups have five. Groups of different sizes may be appropriate for different purposes. There are several factors to consider in determining group size.

One factor is available time allotted for the work. The larger the group, the more time it takes to include everyone's ideas and to reach consensus. The smaller the group, the less time it takes to complete the work and to include everyone; however, fewer ideas may be offered for consideration.

A related factor in determining group size is the level of social skills exhibited by members of the group. Larger groups require higher levels of patience, better listening skills, and the ability to accept different perspectives, while smaller groups can succeed with fewer social skills and lower levels of skills in the group.

Another factor is the willingness of students to engage in group activities. If the class contains many students who tend to hang back and avoid involvement, the best group size is two. It is hard to be left out in a group of two, and students must be more accountable when working in pairs.

Finally, if the group task involves additional complexities or the need to generate a variety of ideas, then larger groups may be best. Students are ready for groups of four or even five if they are willing and able to take the time and if they have the skills to include everyone. One transitional alternative is to have students work in pairs initially and then have each pair share with another pair. If grouping is by 3s and there are extra persons, then one or more groups can be increased to 4.

Heterogeneity The strongest groups contain students who are different from one another in gender, ethnicity, skills and abilities, personalities, socio-economic background, previous school experiences, and so forth. When a heterogeneous group of students work together, they bring different ideas, experiences, and points of view to the group. These differences enrich and strengthen the group discussion.

Methods of Selection Given the above principles for forming groups, there are at least three methods of actually determining which students will be in which groups.

One way to form groups is for you to determine the groupings. You pick individual students to work with other individual students, based on considerations of group size and heterogeneity. However, you should not attempt to create the perfect group by trying to blend all of the criteria at one time. Rather, you should base the groupings on the tasks at hand and on reading skills, where individuals should be grouped so that there is a range of reading skills available in each group. Or, if members of a class of ninth-grade students tend to sit and talk only with students from their own middle school, then the groupings should mix students from different feeder schools.

Another method for determining groups is random selection. While you will want to use a variety of methods during the year, one method is to simply write each student's name on a slip of paper or a popsicle stick, put the items in a container, and then draw out the number of names corresponding to the size of the group predetermined for a particular investigation or lesson. When a random method is used, both teacher and students must be ready for new and possibly untried combinations of students. Students often like random selection best because it seems fair, whereas teacher selection appears to be "fixed."

A third method that generally is not recommended is letting students select the groups themselves. When students pick their own partners, they tend to pick their friends, or people who are like themselves, not different. Some students may not be picked at all. Student selection therefore can violate the basic premises of collaborative learning.

Student selection, however, may be appropriate within limits. For example, you might say, "You may pick your groups today. Find three other students (for a total of four), include males and females, and make sure that you are in a group with students with whom you do not sit at lunch or hang out with after school. No one may be left out." If students can follow these guidelines (or others that you choose), then student-selected groups may be used on occasion.

Student Reaction to Group Compositions No matter which method is used to select the members of each group, the students will have mixed reactions to the groupings. You should start very early to set limits on the acceptable reactions by discussing with the students what they may and may not say when they find out who is in their group. Some possible guidelines include the following:

- Groups will be changed throughout the semester and the year, but they will not be changed until any present problems are resolved.
- You must treat groupmates with respect.

Duration of Group Memberships Because different groups will serve different purposes, the length of time that groups stay together will vary. For example, pairs of students who sit next to each other might check homework at the beginning of the class period for three to five minutes. Two pairs might combine to create a four-person group for investigations. Work groups for projects or investigations might remain the same for each unit or even throughout a grading period. The goal is to have the students work with many different combinations of classmates during the school year.

Dealing with Student Absences "My partner isn't here today. Who do I meet with?" "He has the group paper, but he's gone today!" When a missing group member has materials the group needs, it is difficult for the group to continue its work. When students are absent, they miss the content and the practice provided by group work. Whatever the composition and duration of the groups, absentees can pose problems but they are manageable ones. (Some studies have even indicated that students increase their attendance in classes that use collaborative learning.) Here are some suggestions for dealing with problems related to student absences:

- For long-term groups (two to nine weeks), use groups of four students. With groups of four, it is very likely that there will be at least two or three students in the group on a given day, so the group is still a group, and you can avoid the awkward and time-consuming task of moving a lone person to a new group.
- Do not carry over completed activities into additional class periods to provide catch-up time for students who were absent.
- Encourage each group to be responsible for helping absent members make up missed work by giving them an update by phone before class or immediately upon their return to class.
- For short-term investigations that cover two or more class periods, decide the placement of a returning student on a case-by-case basis. If a student has missed one day of a three-day investigation, the student may be placed in a group. If a student has missed two days of a three-day investigation, the student could be placed in a group for the class period and then be asked to do a shortened investigation addressing the same content, either individually or with the assistance of a classmate, following the class period.
- When a student returning to class does not have an assigned group, place the student in a group. Pick the best group for the student, that is, pick a group that is friendly, patient, and likely to include the student as a working member.
- When a group is preparing one report of their work, do not allow the report to leave the room while still in process. For any continuing activity, have group folders or a group basket ready to retain student work. The next day, the group has its work, regardless of student attendance.

Classroom Techniques for Effective Collaborative Learning

While productive and congenial collaborative learning depends to a significant degree upon student attitudes and skills and upon the composition of groups, much of the success of group learning depends upon the teacher's management of the classroom. Key areas for ensuring effective group work include:

- giving clear directions,
- establishing a materials center,
- setting up the work area for face-to-face interaction,
- establishing classroom routines,
- using a variety of techniques to develop positive interdependence,
- establishing individual accountability, and
- handling conflict effectively.

Giving Clear Directions Explicit, well-organized directions are essential for effective group work. Because of the variety of reading levels and differences in learning styles that may exist within groups, it is helpful to have the directions be visible to students. Reading those directions aloud and checking for understanding before students move into groups helps auditory learners. Keeping the directions displayed during the investigation helps visual learners. If asked a question that is answered in the directions, respond by simply pointing to the display.

Clear directions for problems in *Core-Plus Mathematics* are often provided in the student text; however, if you create or revise directions, care should be taken that those directions are clear. Time spent providing clear directions pays off later.

Establishing a Materials Center Another way to help maximize efficiency in a collaborative learning environment is to establish a materials center, a single location where designated students can get all supplies and handouts for their groups and store their group work. You should place the supplies there before class. Students should get into the routine of picking up all materials from, and returning them to, the materials center. This approach saves time and encourages both you and the students to get and remain organized.

Setting Up for Face-to-Face Interaction The physical location of students in the room affects how they respond to the content of the lesson, to the teacher, and to one another. For group work, students need to be "eye-to-eye" and "knee-to-knee." In other words, students in the same group should face their groupmates and face away from the teacher and other groups. A minimum of furniture is best. Less table surface is needed when students share materials than when all students have their own materials. (See the discussion on "limited" and "jigsawed" materials on pages 48–49 of this guide.) With shared materials, three or four chairs can be clustered around a single desk or at the end of a rectangular table. In any group, students should be close together so that everyone can see easily and can hear without talking loudly.

Establishing Classroom Routines Establishing routines for getting into groups, moving furniture, and becoming quiet all help to facilitate cooperative learning. The Dishon/Wilson O'Leary model recommends the use of the "3 Rs" to teach a routine: reveal, rehearse, and reinforce (Dishon and Wilson O'Leary, 1994). A "Quiet Signal" is one important routine for a collaborative classroom.

1. Reveal: Name the routine, explain why it is needed, tell how it works, and demonstrate the behavior.

   "The routine you will learn today is called the Quiet Signal. It is necessary to have a Quiet Signal because you will be busy working in groups and at some point I will need your attention. I would like to do this in a quiet way so that you can quickly finish your sentence and give me your attention. I will simply raise my hand. If you are not talking and see my hand, continue listening to whomever is speaking and then raise your hand. If you are speaking, finish your sentence and then raise your hand to acknowledge the signal." At this point, you raise a hand.

Berett Wilbur

2. Rehearse: Students practice the procedure while you observe. The students should practice several times under different conditions.

   "Now I would like to have you practice this routine. In a moment, I want you to turn to your neighbor and talk about something you like to do on the weekend. Then listen while your neighbor does the same. You will have two minutes." Move quietly around the room listening while students talk. At the end of two minutes, stand quietly at the front of the room and raise one hand. Students' hands go up slowly at first and then more quickly as more students see the signal. "Thank you. That worked quite well. Remember to finish your sentence before you raise your hand. This time, let's practice with several people walking around the room, some people talking quietly, and the rest working at your desks." This time, let students' movement continue for 30 seconds or so and then raise one hand. "I noticed people seeing my hand and raising one of theirs even when they were out of their seats. That was just fine!"

3. Reinforce: It is important to practice, give feedback, review, and reteach if necessary until students use the routine consistently in appropriate ways. Also, on some occasions after using the routines, be sure to ask students for their perception of how it is working.

   "I want to ask you about how the Quiet Signal is working. Did the person who was speaking when the Quiet Signal was noticed finish his or her sentence?" "What can you do so that the person talking finishes the sentence and doesn't start another one?" Students offer possibilities. The discussion continues and another brief practice session is conducted.

One caution: Be sure that you are using the routine as it was taught. Teachers who have problems with the Quiet Signal often are talking while they have their hand up or are walking around reminding people to be quiet. Teach the routine by standing quietly, not looking directly at any student, and then waiting until it is quiet. Taking the time to teach the routine will be beneficial to you and your students and will facilitate successful collaborative group work.

Developing Positive Interdependence Positive interdependence, the extrinsic conditions that motivate students to work together, is at the heart of collaborative learning. There are various types of tools for building positive interdependence: focusing on group processes, sharing materials, and sharing roles. At least one of these tools must be used during any collaborative learning lesson, although more can be included.

Focusing on Group Processes—The most obvious tools for ensuring that students work together in their groups have to do with focusing on group processes. For example, the procedures described for giving directions clearly focus on the group and reinforce the principle of group structure.

The practice of answering only group questions is a powerful tool for developing positive interdependence. When students are working in a group, you should establish the general practice of answering only those questions that have been decided upon by the group. If a student has a question about the group's work, the group should be consulted. If no one has a response that helps, or if there are various responses and an agreement cannot be reached, this group of students may ask you. When the group has a question, all members must raise a hand. This signals to you that everyone agrees that help from you is needed. It should be understood by all that, when you arrive at the group, anyone in the group may be called on, not just the person who asked the original question. If the person called on does not know the question, you may say, "I will come back when you have a group question." If the person called on knows the question, you may ask probing questions to assess student understanding or prompt thinking, give a hint to help the group determine an appropriate answer, refer them to another group, or answer the question.

Before beginning group work, it would be helpful for students to understand the reasons for responding only to group questions. The belief behind the practice of the group question is that the group is a main resource for students. It is important for students to realize that they can think and solve problems within their groups. The teacher's job is to facilitate group work, not to participate as a member of any one group.

Establishing a group goal is another way to help students achieve positive interdependence within their groups. One such goal might be to work through an investigation or to prepare for a class discussion of a Summarize the Mathematics.

Still another way to increase the likelihood that students will think, talk, and work together in a group is to have the group create a single group product. The one product, which must be specifically described in the directions for the problem, is worked on by everyone in the group. Everyone must give ideas or opinions; everyone must write, draw, or use technology tools when agreed to by the group. This product is not created in an assembly line, like an add-a-line poem; rather, it is shared, but not divided, equally among the group members.

Sharing Materials—Another tool for developing positive interdependence is sharing materials in one of two ways. With "limited" materials, the group is limited to one set of materials. If the problems require a ruler, marker, scale, and technology, there is only one of each item in each group. These materials then rotate among the members of the group, with each student taking a turn measuring, writing, weighing, or calculating, at the direction of the group.

With "jigsawed" materials, the different items needed for the activity are distributed among the group members. In other words, everyone in the group has something that the other group members need. If measurements are required, one person may hold the item to be measured while others do the measurement and check each other's measurements by re-measuring. Or each student may have a different piece of information, and the group needs all the pieces of information to respond to the problem at hand.

Sharing materials in these ways helps ensure that the quiet, shy, or inactive student is included in the task, and the sharing also helps reduce the influence of dominating students who want to use only their own ideas. Because individual students who have their own materials have less reason to interact with other group members, they are more likely simply to socialize in their talking or not talk at all. Sharing materials thus encourages students to engage in conversation about the work, to explain, predict, negotiate, and reach consensus.

Sharing Roles—Sharing roles is also a type of tool to develop interdependence within groups, and there are several approaches to the assignment of roles. In what Dishon and Wilson O'Leary call "specifically assigned rotated roles," each student in the group has a particular function for a particular group session. This method of assignment can be seen in the references to the roles of Experimenter, Recorder, and Quality Controller in Unit 1 of Course 1. Depending upon the circumstances and the task at hand, of course, there can be different combinations of roles. For example, when students work in groups of three or four, the roles might be selected from Reader, Recorder, Quality Controller, Coordinator, and Reporter. With this type of role sharing, you may rotate the job assignments each class period or before the beginning of each new investigation.

In another type of role sharing, everyone is responsible for taking a turn at each job during group work time. For example, each student takes a turn being the measurer or being the writer. Then when it is time to report on group work, each student does part of the reporting. All group members are thus responsible for contributing ideas and information, as well as for checking the accuracy of their work.

Still another way to deal with roles is not to assign them at all. Instead, after the collaboration is completed, ask students to reflect on the tasks that had to be done and how these tasks were handled.

Establishing Individual Accountability While the use of the techniques described above for development of positive interdependence is at the heart of collaborative learning, establishing individual accountability is also a very important component of collaborative learning.

One way to start developing accountability is to require individual student signatures on group products. At the end of work time, each student signs the group product with full name, not scrawled initials. A signature means, "I helped and I agree." This declaration of ownership and participation helps formalize the importance of being involved and being responsible for what the group discusses, decides, creates, agrees upon, and learns. Such visible ownership of the group product also decreases the feeling of anonymity that can accompany group work and begins the accountability process. In addition, the signature is a tool to use if a student says later, "Well, I didn't agree with what my group did." In such a case, the teacher responds, "Isn't this your signature? When you sign, you become responsible."

In some instances, of course, groups cannot reach consensus. Students may then designate and sign the parts to which they do agree, and indicate where and how they disagree.

Another fundamental way to ensure individual accountability is by holding every student responsible for being able to describe any aspect of the group activity. For example, you may decide not to select students for the role of reporter until after the investigation has been completed. In this way, all students must be ready to report on their group's findings by describing the group product, by explaining why it looks the way it does, by giving reasons for the group response, and so on.

When designating who will be the reporters, you may randomly select one person to report from each group. Another method of selecting involves the idea of shared roles: you randomly select a group or groups to report and have everyone in the group present part of the report.

No matter how the reporters are selected, to be fair and to motivate the students to pay attention to the task at hand, the method of selection, but not the identity of the reporters, should be announced before the investigation begins.

Opportunities for individual accountability are an integral part of each lesson in *Core-Plus Mathematics*. For example, at the Summarize the Mathematics, some students are called upon to report for their groups; random selection of reporters works well here. Following each Summarize the Mathematics, another opportunity for individual accountability is provided by the Check Your Understanding task. After developing the mathematical ideas in groups and then sharing, refining, and summarizing them in a class discussion, students individually complete the Check Your Understanding task based on their understanding of the mathematics. Their work on this task can be evaluated and recorded in a gradebook. This kind of accountability should also be decided upon and announced before work begins.

When students work in groups, it can be difficult for the teacher to know what an individual student participated in or learned. By including individual accountability in every investigation, the teacher alerts the students to the fact that they will be held accountable, and thus increases the likelihood that students will pay attention to, and participate in, group work.

Handling Conflict Effectively A final area in which effective classroom management contributes to the collaborative learning process is in the handling of counterproductive conflict. The key to preventing such conflict lies in adherence to the norms for promoting respect. Even when rules for acceptable behavior have been discussed and posted, conflict can arise during group work. The amount of potential conflict is related to students' social skills. Students with a high degree of social skills will discuss, negotiate, and reach consensus with minimal conflict. When presented with new or different ideas, students with few social skills may use group work as a time to argue. The goal is to encourage healthy debate and to eliminate negative conflict.

To reduce further the likelihood of negative conflict, teachers may set the stage for the investigation by initiating a discussion to check for understanding or by conducting a brainstorming session (without evaluation of the ideas contributed). The Think About This Situation that begins each lesson serves this purpose.

Another way of preparing students to handle conflict is to have them complete and then discuss prompts such as, "When we had a disagreement, it was helpful to … ." Roleplaying can also help students understand the differences between healthy debate and negative conflict and help them practice handling conflict.

When a group seems to be at an impasse, you should help the group resolve a conflict rather than simply break up the group. In resolving conflict, as in other aspects of collaborative learning, the teacher functions as guide or coach. Giving help does not mean entering into the conflict or solving the problem for the students. You must help the students to become their own problem solvers.

Conflict can be healthy and can stimulate thinking if teachers help students learn how to question each other with a positive tone; however, conflict resolution requires complex skills and will take time for students to practice and learn.

Conclusion

An atmosphere in which students feel free to express their thoughts without derision from classmates, are encouraged to think deeply about mathematics, and are provided the opportunity to grow intellectually is truly a challenge to create. The culture created within the classroom is crucial to the success of the *Core-Plus Mathematics* program. It is important to inculcate in students a sense of inquiry and responsibility for their own learning. Without this commitment, active, collaborative learning by students cannot be effective. The percentage of ninth-grade students who have already attained these skills may be small. The work ethic and social skills of ninth-graders are not always ideal in many cases, but teachers indicate that, with persistence, this changes. In some cases, students returning to school as tenth-graders seem to have miraculously matured over the summer and are actually eager to work collaboratively with their classmates.

In order for students to work collaboratively, they must be able to understand the value of working together. Some students seem satisfied with the rationale that this is important in the business world. Others may need to understand that the struggle with verbalizing their thinking, listening to others' thinking, questioning themselves and other group members, and coming to an agreement increases their understanding and retention of the mathematics and contributes to forming important thinking skills or habits of mind.

This level of understanding of the importance of collaborative groups is particularly important for students who are inclined to quickly and superficially cover the mathematics and then move on. They often resist thinking deeply about the mathematics and fail to see the value in a multiple-perspective approach. Once these students understand that collaborative work broadens and deepens their own mathematical understanding, they can exert a positive influence on their classmates.

Issues involved in helping students to work collaboratively will become less pressing as both you and your students gain experience in this type of learning. You may find it helpful to refer back to this guide and discuss effective collaborative group strategies with your colleagues a few weeks into the semester.

Equity and Access for All Students

One question frequently asked by districts adopting *Core-Plus Mathematics* is related to equity and approaches to accommodate the program for underrepresented students. Mixed-ability classes, a focus on problem-solving, high expectations for all students, attention to a broad array of mathematical topics, and allowing students to restate problems in their own words also appear to help students from different racial, ethnic, and linguistic groups be more successful in mathematics. In addition, several research studies have provided evidence that introducing activities through classroom discussions, teaching students to explain and justify their thinking, and making real-world contexts accessible to students promote greater access and equity in mathematics classrooms. Practices that help promote equity are briefly discussed below.

Introducing Investigations Through Class Discussions

Group and class discussions regarding the aim of investigations, the meaning of contexts, the challenging points within problems, and possible problem access points to which students might turn make tasks more evenly accessible to students. In cases where students use informal or non-mathematical language to explain their reasoning, the teacher may consider rephrasing or re-voicing the student's explanation, using more formal mathematics language.

Teaching Students to Explain and Justify Their Thinking

Giving explicit attention to explaining thinking and evaluating what makes a good piece of work helps students improve their work. Explicit attention should include having students think about and discuss how a given solution differs from others' solutions, make connections between the methods by indicating points of agreement and disagreement, and explain why they selected their particular methods.

Making Real-World Contexts Accessible

Considering the constraints that real situations involve and connecting these situations with issues and topics in their own lives helps students view mathematics as something that will help them interpret their world. The focus of class discussion should aim at supporting students' understanding of the culturally relevant suppositions intrinsic to a problem context and the development of imagery of key mathematical relationships described in a task.

Access to Technology

The use of graphing calculators and computer software is beneficial for many special needs students. Using technology allows students to make more extensive use of multiple representations without needing to construct them by hand. It may be the case that some students do not have access to *CPMP-Tools* software or to handheld technology for homework tasks. Approaches taken by schools using the CPMP program to reduce this inequity include providing multiple locations for students to complete homework during the school day. *CPMP-Tools* should be available in classrooms used by special education teachers or other professionals who are assisting students with homework. For homes that do not have Internet access, but have computers, schools provide the software on a USB drive for students to download to their home computer. Technology Tips are available in the *Unit Resource Masters* to assist students who have difficulty retaining methods.

Other Practices that Promote Equity

Some instructional strategies that are helpful for making mathematics more accessible for a diverse population of students, including students with special needs and English language learners, are listed below:

- Use engaging and meaningful contexts.
- Use multiple representations.
- Sequence instruction to move from concrete to representational to abstract (from specific to the general).
- Offer manipulatives.
- Provide examples and nonexamples.
- Offer templates and graphic organizers.
- Use modeling.
- Use cooperative group work.
- Teach metacognitive and problem-solving strategies.
- Provide opportunities for students to build on their prior knowledge and experiences.
- Immerse students in the language of mathematics.
- Provide opportunities for guided and independent practice.
- Use frequent assessments.
- Provide timely and constructive feedback.
- Have students create their own resources.
- Use organizational systems for notebooks/binders.
- Reduce amount of copying for students.
- Adjust time for tasks and pacing.
- Adjust amount of work.
- Help students to become independent learners.

Core-Plus Mathematics field-test materials were reviewed by the Educational Development Center, Inc. (EDC) through an accessibility lens in order to identify strengths and potential barriers for students with special needs. EDC found that many of the above strategies were already an integral part of the materials. In particular, the following features of *Core-Plus Mathematics* improve access for all students.

Focus Questions Focus questions at the beginning of each investigation provide students with the goal(s) of the investigation. By having this identified at the beginning of the investigation, students see the underlying mathematical question(s) of the investigation.

INVESTIGATION 1

Predicting Population Change

If you study trends in population data over time, you will often find patterns that suggest ways to predict change in the future. There are several ways that algebraic rules can be used to explain and extend such patterns of change over time. As you work on the problems of this investigation, look for an answer to this question:

What data and calculations are needed to predict human and animal populations into the future?

MATH TOOLKIT

Some students may wish to add an example of a *NOW-NEXT* rule and the calculator procedure for using the ENTER key to find values recursively.

Mathematics Toolkit Having a mathematics toolkit is an excellent accessibility strategy. It is important for special needs students to create their own resources that they can use. The toolkit could be particularly helpful for students with memory difficulties. It has the added benefit of helping students to become more independent.

Assessment Strategies Ways to differentiate assessment include the following:

- Use electronic versions of the assessments and provide questions or hints for selected students.
- Provide dual-language exams where possible.
- Offer alternatives, for example, a student could draw a diagram instead of writing an explanation.
- Provide additional time to complete assessments.
- Use many strategies suggested for assessment of exceptional education students:
  - enlarge font size,
  - avoid visual crowding,
  - include an organized, adequate work area,
  - scaffold the meaning of the question by including the T of a required T-chart or providing a graph grid,
  - avoid idioms such as “best buy,” and
  - selectively use a highlighter on the English language learner students’ tests.

Formative and Summative Assessments

Throughout the *Core-Plus Mathematics* curriculum, the term *assessment* is meant to include all instances of gathering information about students' levels of understanding of and their disposition toward mathematics for purposes of making decisions about instruction. The dimensions of student performance that are assessed in this curriculum (see chart below) are consistent with the assessment recommendations of the National Council of Teachers of Mathematics' *Assessment Standards for School Mathematics* (NCTM, 1995). These recommendations are much broader than those of a typical testing program.

Assessment Dimensions

Process	Content	Attitude
Problem Solving Reasoning Communication Connections	Concepts Applications Mathematical Representations Procedures	Beliefs Perseverance Confidence Enthusiasm

Sources of Assessment Information

Several kinds of assessment are available to teachers using *Core-Plus Mathematics*. Some of these sources reside within the student text itself, some of them are student-generated, and some are supplementary materials designed specifically for assessment. Understanding the nature of these sources is a prerequisite for selecting assessment tools, establishing guidelines on how to score assessments, making judgments about what students know and are able to do, and assigning grades.

Curriculum Sources Two features of the curriculum, questioning and observation by the teacher, provide essential and particularly useful ways of gathering assessment information.

Questions and Questioning—The student texts use questions to facilitate student understanding of new concepts, how these concepts fit with earlier ideas and with one another, and how they can be applied in problem situations. Whether students are working individually or in groups, the teacher is given a window to watch how the students think about and apply mathematics as they attempt to answer the questions posed by the curriculum materials. In fact, by observing how students respond to the curriculum-embedded questions, the teacher can assess student performance across all process, content, and attitude dimensions described in the chart above.

Student responses to questions such as "What if ... ?" and "Why?" along with their explanations and justifications provide insights into how students reason about and communicate mathematics. How well students see connections among mathematical ideas and their applications can be assessed with questions such as "How is this like [an earlier idea] and how is it different?" A question such as "What do you predict will happen?" may provide insights not only into students' understanding of the relevant content but also into their dispositions toward mathematics. These are just some of the types of questions that can provide good assessment information to help teachers make appropriate instructional decisions. Although such questions are commonplace in the instructional materials, teachers can frame similar questions in order to further probe student understanding. While observing individual students as they respond to such questions, teachers often take notes or complete checklists to help judge the growth of individual students or to provide detailed information for grade reports.

Specific features in the student material that focus on different ways students respond to questions are the Summarize the Mathematics, Check Your Understanding, and On Your Own homework sets. The Summarize the Mathematics sections are intended to bring students together, usually after they have been working in small groups, so they may share and discuss the progress each group has made during a sequence of related problems. The questions in the Summarize the Mathematics are focused on mathematical concepts and methods developed in the investigation. They should help the teacher and student identify and formalize the key ideas of the investigation. Each Summarize the Mathematics is intended to be a whole-class discussion, so it should provide an opportunity for teachers to informally assess the levels of understanding that the various groups of students have reached.

Following each Summarize the Mathematics, the Check Your Understanding tasks are meant to be completed by students working individually. Student responses to these tasks provide an opportunity for teachers to assess the level of understanding of each student.

The homework tasks from the On Your Own sets serve many purposes, including post-investigation assessment. Each type of task in the On Your Own homework sets has a different instructional purpose. Applications tasks provide opportunities for students to demonstrate how well they understand and can use the ideas they learned in the investigations of the lesson. Work on Connections tasks demonstrates how well the students understand links between mathematical topics they studied in the lesson and their ability to connect those topics with other mathematics that they know. Reflections tasks provide insight into students' mathematical thinking and strategic competence. Extensions tasks reveal how well students are able to extend the present content beyond the level addressed in the investigations. The Review tasks allow for pre-assessment of students' understanding of ideas or procedures needed in the upcoming lessons and also provide information on how well students are retaining previously learned mathematics. The performance of students or groups of students on each of these types of tasks provides the teacher with further information to help assess each student's evolving ability to use, connect, and extend the mathematics of the lesson.

Finally, an opportunity for group self-assessment is provided in the last element of each unit, the Looking Back lesson. These tasks help students pull together and demonstrate what they have learned in the unit and, at the same time, provide helpful review and confidence-building for students.

Additionally, in the *Unit Resource Masters* for Courses 1–3, there are Practicing for Standardized Tests masters, and in Course 4, there are Preparing for Undergraduate Mathematics Placement (PUMP) exercise sets. Each PUMP set provides practice and assessment of skills and reasoning techniques commonly assessed on college mathematics placement tests.

Observation–The other fundamental method of assessment that is built into *Core-Plus Mathematics* is observation by the teacher. Teachers, of course, have always learned a great deal by observing individual students as they do mathematics. Such observation continues to be a valuable source of assessment information. Because many problems are completed by students working in small groups, teachers using these materials have many opportunities to observe the performance of students as they work with others.

While students are working on tasks from the investigations, the teacher might listen for evidence of student progress in understanding and applying the mathematical content of the lessons. Before students begin their work, the teacher should identify several tasks that will help determine the progress that students have made in understanding the key mathematical ideas or procedures of the investigation. Then, while observing each group, the teacher can focus on students' work on those items. Assessing student thinking by listening and questioning students as they work through an investigation will also allow the teacher to identify and address misconceptions as they arise.

In addition to developing content knowledge, *Core-Plus Mathematics* provides students an opportunity to develop skill in working with others. In today's society, working well with others in problem-solving groups is important. To help develop these skills, teachers should observe and assess behaviors of group members such as those listed below and provide students feedback.

- Communicating mathematical ideas to other members of the group
- Dividing a task fairly among group members
- Agreeing on a structure for completing a task
- Taking time to ensure that all group members understand
- Recording results regularly and clearly
- Soliciting and using, in appropriate ways, the suggestions and ideas of all group members
- Fairly representing a group consensus
- Pushing the group to think deeply about the mathematics under investigation
- Reporting the group's progress to the whole class in a complete and interesting way

Student-Generated Sources Other possible sources of assessment information are writings and materials produced by students in the form of Mathematics Toolkits, unit summaries, journals, and portfolios.

Mathematics Toolkits–Each student should create a Mathematics Toolkit that organizes important class-generated ideas and selected Checkpoint responses as they complete investigations. Constructing a Math Toolkit prompts are provided in the *Teacher's Guide* to assist in identifying key concepts and methods as they are developed by students.

MATH TOOLKIT

Give a graphical example of a function and a relationship that is not a function. For the function example, use function notation to write a rule that represents your example. Then, describe the theoretical domain and range.

Berett Wilbur

Unit Summaries—A summary template intended to help students organize and record the main ideas learned in the unit is provided in the *Unit Resource Masters*. The synthesis of ideas that occurs during completion of the Looking Back lesson and the final unit Summarize the Mathematics discussion should provide the background for student completion of the unit summary.

Journals—Student journals are notebooks in which students are encouraged to write (briefly, but frequently) their personal reflections concerning the class, the mathematics they are learning, and their progress. These journals are an excellent way for the teacher to gain insights into how individual students are feeling about the class, what they do and do not understand, and what some of their particular learning difficulties are. This information can be very useful for planning instruction that will meet the needs of individual students in the class. For many students, the journal is a non-threatening way to communicate with the teacher about matters that may be too difficult or too time-consuming to talk about directly. Journals also encourage students to assess their own understanding of, and feelings about, the mathematics they are studying.

One effective approach to the use of journals is for the teacher to provide prompts (questions or statements) to which the students give a written response in their journals. Such prompts may be given one or two times a week, and the writing may be done during the last few minutes of class or at home. The best journal prompts are open-ended and encourage students to reflect on the mathematics they have been doing and learning or to reflect on the learning process. Some possible prompts are given below.

- The key idea of the lesson today was …
- What questions were still unanswered at the end of class today?
- Find something that you learned today that is connected to something that you already know. Write about how the two things are connected.
- How do you feel about sharing your work with the class?
- My three personal goals in math this term are …
- When I study for a test, I …
- Describe one thing you did well today and one thing upon which you could improve.
- The problem-solving techniques that I used today were …
- Describe a way in which you helped a classmate today or a way in which a classmate helped you.
- When you get stuck on a problem, what are some things you try in order to get unstuck?
- Identify a task that was difficult (easy) for you today. Describe why it was difficult (easy).
- In what ways did technology help you to better understand the big ideas in today's lesson?

The teacher should collect, read, and respond to each journal regularly. Teacher responses to the journal entries should be nonjudgmental statements of interest or questions seeking clarification. Many teachers stagger their journal collections, reviewing perhaps one-fourth of them each week. Teachers are encouraged to have their students keep journals and write in them regularly.

Portfolios—A portfolio is a collection of a student's work accumulated over time. Typically, portfolios provide a tool for assessing one or more of the following outcomes: student thinking, growth over time, mathematical connections, a student's views on herself or himself as a mathematician, and the problem-solving process as employed by the student. An excellent, practical source about the use of portfolios in mathematics is the NCTM publication, *Mathematics Assessment: A Practical Handbook for Grades 9–12* (1999).

The *Core-Plus Mathematics* assessment program provides many items that would be appropriate for a student's portfolio, including reports of individual and group projects, Mathematics Toolkits or journal entries, teacher-completed observation checklists, and end-of-unit assessments, especially the take-home tasks and projects. One way students can develop a portfolio is to collect all of their written work in a folder, sometimes called a "working portfolio." Then, at least once each semester, students can go through their working portfolios and choose items that they think best represent their growth during that time period. After writing a paragraph or two explaining why each piece of work was chosen, each student can place the chosen items with the written rationales into a new folder that becomes the actual portfolio.

Curriculum-Provided Assessments The *Core-Plus Mathematics Unit Resource Masters* and eAssessment provide lesson quizzes, tests, and additional banks of items for forming semester exams. Lesson quizzes are intended as formative written assessments. Summative assessments include tests, take-home tasks, and projects. In some cases, educators use student-written tests as formative assessments by providing individual students the opportunity to re-learn and re-test on selected topics as needed.

Unit Resource Masters—The *Unit Resource Masters* are an additional source of assessment information. They include lesson quizzes and unit assessments in the form of tests, take-home tasks, and projects. There are also banks of questions and projects from which you can form end-of-semester exams following the Unit 4 and Unit 8 assessment masters.

The online eAssessments for *Core-Plus Mathematics* allow teachers to modify the curriculum-provided assessment items or to create formative or summative assessments using a combination of curriculum-supplied items and ones written by the teacher.

For most Project-developed assessments, calculators are intended to be available to students. Teacher discretion should be used regarding student access to their textbook and Math Toolkit for assessments. In general, if the goals to be assessed are problem solving and reasoning, while memory of facts and procedural skill are of less interest, resources may be allowed. However, if automaticity of procedures or unaided recall are being assessed, it is appropriate to prohibit resource materials.

Lesson Quizzes–Two forms of a quiz covering the main ideas of each lesson are provided. These quizzes are comprised of problems meant to determine if students have developed understanding of the important concepts and procedures of each lesson. The two forms of each quiz are not necessarily equivalent, although they assess essentially the same mathematical ideas. Since many rich opportunities for assessing students are embedded in the curriculum itself, you may choose not to use a quiz at the end of every lesson.

Unit Tests–Two forms of tests are provided for each unit and are intended to be completed in a 50-minute class period. The two forms of each test are not necessarily equivalent, although they assess essentially the same mathematical ideas. Teachers should preview the two versions carefully to be sure that the unit assessment aligns with the learning goals emphasized.

Take-Home Assessments–Take-home assessment tasks are included for each unit. The students or the teacher should choose one or, at most, two of these tasks. These assessments, some of which are best done by students working in pairs or small groups, provide students with the opportunity to organize the information from the completed unit, to work with another student or group of students, to engage in in-depth problem solving, to grapple with new and more complex situations related to the mathematics of the unit, and to avoid the time pressures often generated by in-class exams. These problems may also require more extensive use of technology than is often available in the regular classroom during testing situations. You may wish to use these more in-depth problems as a replacement for a portion of an in-class end-of-unit exam.

Projects–Assessment traditionally has been based on evaluating work that students have completed in a very short time period and under restricted conditions. Some assessment, however, should involve work done over a longer time period and with the aid of resources. Thus, assessment projects are included in unit assessments. These projects, which are intended to be completed by small groups of students, provide an opportunity for students to conduct an investigation that extends and applies the main ideas from the unit and to write a summary of their findings. Many of these might also allow for students to present their work in a variety of ways. You may have students who would rather prepare and present their work orally or visually using computers and/or video equipment. In this way, the projects can provide an opportunity for students to use their creativity while demonstrating their understanding of mathematics.

Midterm and Final Assessments–A bank of assessment tasks, from which to construct midterm and final exams that fit your particular class needs and emphases, are provided with the Unit 4 and Unit 8 *Unit Resource Masters*. In addition to problems similar in form to those on the quizzes and tests, these assessment banks include several multiple-choice problems for each unit.

Extended assessment projects are also included with the end-of-year assessments. These projects are investigations that make use of many of the main ideas encountered in the curriculum. They require use of material from more than one unit. The projects are intended to be completed by small groups of students working over a period of time. You may wish to have different groups work on different projects and then give presentations or create posters of their work.

Scoring of Assessments

High expectations of the quality of students' written work will encourage students to reach their potential. You will notice the solutions will sometimes indicate that "responses may vary." This is used when more than one correct response is acceptable. Students should always have reasonable explanations for their responses. The quality of the responses will vary and some responses may simply be incorrect. Some teachers use quality student responses as model responses for the class throughout the course. Continued emphasis on reasonable and clear explanations helps students develop their thinking and communication skills throughout their mathematics studies.

Assigning scores to open-ended assessments and to observations of students' performance requires more subjective judgment by the teacher than does grading short-answer or multiple-choice tests. It is therefore not possible to provide a complete set of explicit guidelines for scoring open-ended assessment items and written or oral reports. However, the following general guidelines may be helpful (adapted from *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions*, National Council of Teachers of Mathematics, 1991). Teachers can use this general framework to develop guidelines for specific assessment items.

Open-Ended Items When scoring student work on open-ended assessment tasks, the goal is to reward, in a fair and consistent way, the kinds of thinking and understanding that the task is meant to measure. Preparing to score open-ended assessment tasks is best done using a three-step process. First, teachers should have a general rubric, or scoring scheme, with several response levels in mind. As the name suggests, the general rubric is the foundation for scoring across a wide range of types of open-ended tasks. The following general rubric can be used for most assessment tasks provided with *Core-Plus Mathematics* materials.

To score a particular task in a reasonably unambiguous, fair way, the teacher will need a specific rubric, which is the result of rewriting the definitions of the scoring levels in the general rubric so that they explicitly account for the details of the assessment task at hand and apply to the range of actual student responses to that task. (As an illustration, consider the sample task on page 62.) This rubric should take into consideration the ways in which students construct their responses to meet the expectations of that task. The specific rubric is the second step in the three-step process.

Finally, since verbal definitions for score levels can never be entirely unambiguous, the teacher will need to identify examples of student work to anchor the specific rubric. These examples are often referred to as anchor items. The anchor items for each scoring level should be chosen to illustrate the range of responses that are typical of students who score at that level. On the next three pages, the complete process for scoring is described using a sample open-ended task and examples of actual student work for *Core-Plus Mathematics*.

General Scoring Rubric

For most open-ended mathematics tasks, credit can be assigned in decreasing amounts according to the following levels of performance. This general rubric has five scoring levels corresponding respectively to 0 to 4 points, but three or four levels may work better for some tasks. Not all parts of the description of each level below will be relevant to every open-ended task.

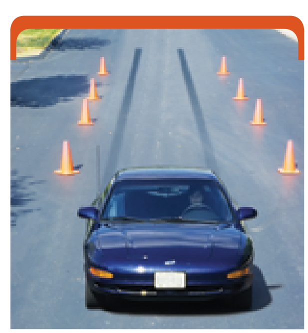
4 Points	Contains complete response with clear, coherent, and unambiguous explanation; includes clear and simple diagram, if appropriate; communicates effectively to identified audience; shows understanding of task's mathematical ideas and processes; identifies all important elements of task; includes examples and counterexamples; gives strong supporting arguments
3 Points	Contains good solid response with some, but not all, of the characteristics above; explains less completely; may include minor error of execution but not of understanding
2 Points	Contains complete response, but explanation is muddled; presents incomplete arguments; includes diagrams that are inappropriate or unclear, or fails to provide a diagram when it would be appropriate; indicates some understanding of mathematical ideas, but in an unclear way; shows clear evidence of understanding some important ideas while also making one or more fundamental, specific errors
1 Point	Omits parts of question and response; has major errors; uses inappropriate strategies
0 Points	No response; frivolous or irrelevant response

Sample Assessment Task

6 The stopping distance d in feet for a car traveling at a speed of s miles per hour depends on car and road conditions. Here are two possible stopping distance formulas: $d = 3s$ and $d = 0.05s^2 + s$.

a. Write and solve an equation to answer the question, "For what speed(s) do the two functions predict the same stopping distance?" Illustrate your answer with a sketch of the graphs of the two functions, labeling key point(s) with their coordinates.

b. In what ways are the patterns of change in stopping distance predicted by the two functions as speed increases similar and in what ways are they different? How do the function graphs illustrate the patterns you notice?



Doug Martin

The process of developing a specific rubric for this task involves an initial decision concerning whether to score the entire task as a unit or to score each part separately. In this case, it could be reasoned that the parts are sufficiently different to make scoring each part the better decision. Following that decision, consider Part a by first asking what a top-level (4-point) response should contain. After specifying the characteristics of a solution required for 4 points, next ask what a minimal solution must contain; that is, what must the response include for a score of 1 rather than a 0. Identifying both the "best" and "least" acceptable solution can usually be done without reference to student work, although some minor changes in these initial definitions may need to be made later to account for some unanticipated student responses.

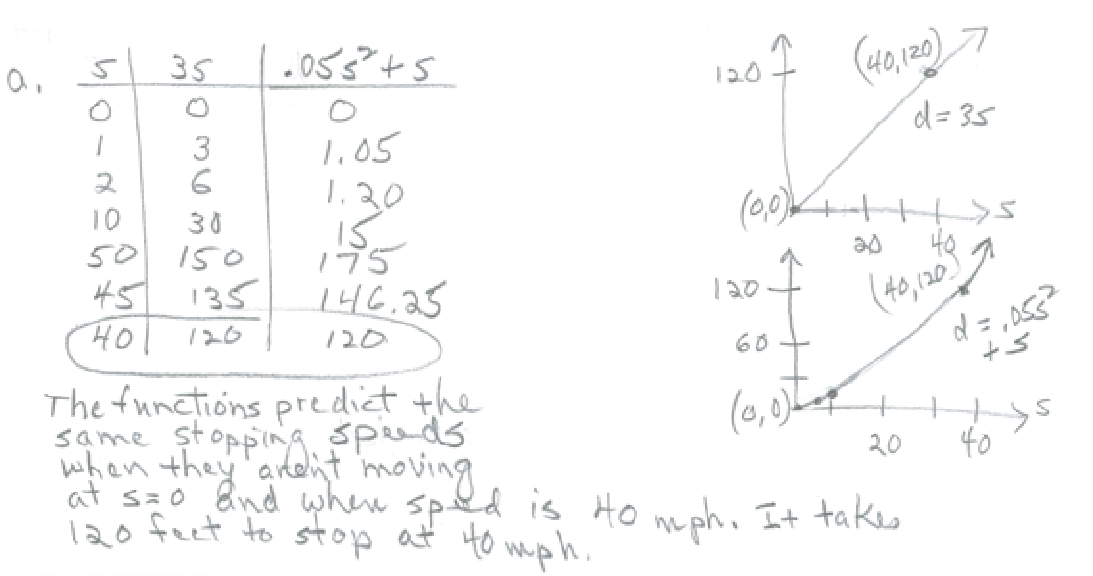
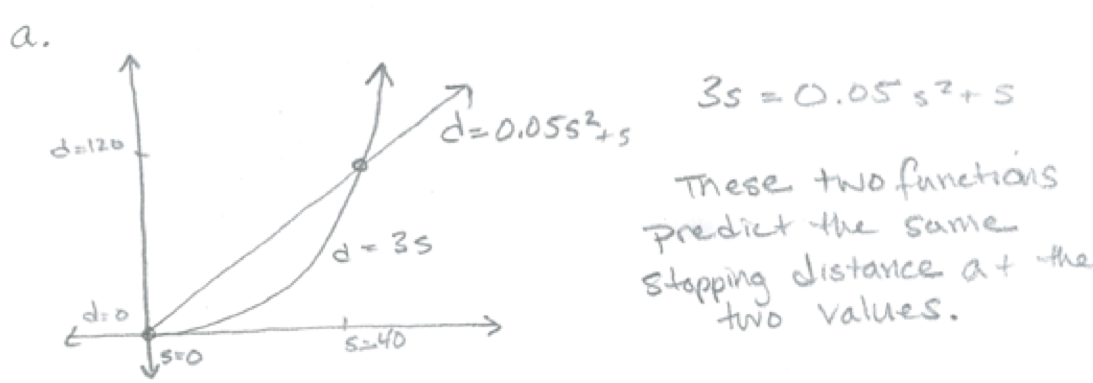
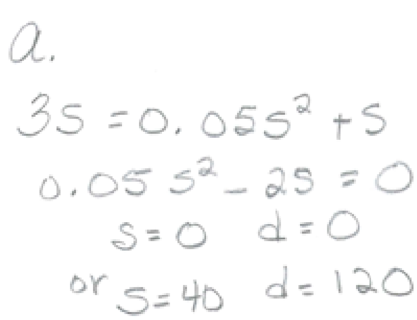
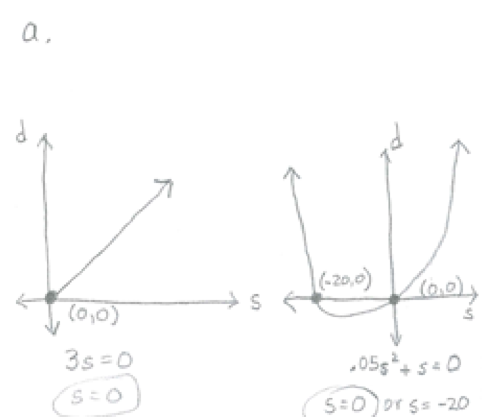
Distinctions between scores of 4 and 3, 3 and 2, and 2 and 1 are more difficult. It is helpful to make an initial attempt at defining scores of 3 and 2, using only an analysis of the item and what are likely responses, but then to move quickly to examining a set of student work. Actual student responses help refine definitions, forcing decisions about what should count in each scoring category. Refinements can be made to account for unexpected student responses, while keeping in mind the general rubric and the ultimate goal: to reward in a fair and consistent way the kinds of thinking and understanding that the task is meant to measure.

By repeating the process described in the previous paragraph for Parts b–d, a specific rubric similar to the following could be constructed. Recall that a score of 0 is given for either no response or for a completely irrelevant response.

Specific Scoring Rubric

Part	4 points	3 points	2 points	1 point
a	Writes equation $3s = 0.05s^2 + s$ correctly. Describes and justifies solution method such as using graphs, tables, or symbols to determine that when speeds are 0 or 40 mph, both functions predict same stopping distances of 0 and 120 ft. Provides sketches of the graphs of $d = 3s$ and $d = 0.05s^2 + s$ labeling coordinates (0, 0) and (40, 120).	Provides correct answer but contains execution errors in writing equation, solution method (using tables, graphs, or symbols), sketches of graphs, or labeling key points with coordinates.	Incorrect answer with correct equation, but work contains major errors in solution method (e.g., algebraic error not just arithmetic), fails to provide diagram, or does not include coordinates for key points.	Incorrect answer with wrong equation, inappropriate work, or no work.
b	Correct answer similar to: "For speeds greater than zero, both graphs are increasing. However, the function $d = 0.05s^2 + s$ is increasing at an increasing rate while $d = 3s$ increases at a constant rate. The graphs illustrate the patterns for these quadratic ($d = 0.05s^2 + s$) and linear ($d = 3s$) functions."	Right idea, namely that both functions increase, yet at different rates. But fails to correctly describe this pattern by missing labels (e.g., after 0 mph) or unclear which function increases at either a constant ($d = 3s$) or increasing rate ($d = 0.05s^2 + s$). Makes connection to graph illustration.	Correctly states at least either how the two functions are similar (both functions increase after 0 mph) or how they are different (increases at an increasing rate for $d = 0.05s^2 + s$ and at a constant rate for $d = 3s$) but fails to include both or provides error in describing one of the patterns illustrated in the graph.	Incorrect response that shows some relevance to patterns of change predicted by the two functions or illustrated in the function graphs.

Anchor Items

Score	Student Responses for Part a
4	
3	
2	
1	

Shown on the previous page are four examples that correspond to scores of 1–4 that are meant to clarify the rubric for Part a. In the first example, the student applies numerical thinking using tables of values of *s* and *d* for both functions. Using the table, the student graphs both functions on separate sets of axes. The student writes the correct equation and determines that the two functions predict the same stopping speeds of 0 mph and 40 mph. Although the graphs are shown on separate axes, complete understanding seems evident. (The teacher might encourage this student to consider when it is valuable for visual thinking to graph more than one function on the same axis.)

The second example provides a correct answer with errors in executing and communicating the work. The student work suggests a graphical or technological approach to solving the task. Although the student provides sketches of both functions on the same set of axes, the student mislabels the functions in the graphical representation. The student determines the correct answer, however, provides a weak written statement by not interpreting the coordinates in context.

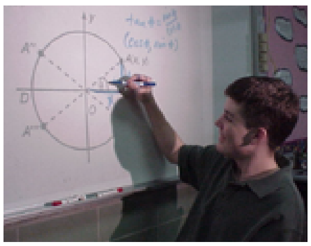
The third example provides student work for solving the problem algebraically, however, fails to provide a sketch of the graph of the two functions.

The fourth example sketches the graph of the two functions on separate sets of axes and produces two equations, $0 = 3s$ and $0 = 0.05s^2 + s$. The work suggests that the student is solving a related problem, namely, what is a common stopping speed that is predicted by each of the respective two functions.

Anchor Items

Score	Student Responses to Part b
4	b. $d=3s$ and $d=0.05s^2+s$ both increase when speed is greater than zero. $d=3s$ graph increases at a constant rate and $d=0.05s^2+s$ increases at an increasing rate.
3	b. Both graphs increase when $s>0$. One increases more than the other then they switch at 40mph.
2	b. Both graphs increase for speeds larger than zero. But $d=.05s^2+s$ increases more quickly than $d=3s$ because $d=3s$ has only a contant rate.
1	b. Both are similar in predicting the same stopping distance at speed 40. They differ for all other values.

Note that a score of 4 requires explicit mention that the linear function increases at a constant rate and that the quadratic function increases at an increasing rate. The response that was scored 3 suggests that the student may have had the correct idea, but failed to express it explicitly. The response in the 2-point category was an incomplete response in comparing the two functions, but at least it identifies that both graphs increase for speeds larger than zero and that the linear function has a constant rate. The response assigned a score of 1 failed to describe the two functions beyond one of the common stopping speeds (40 mph).



Reports and Presentations Four or five levels of holistic scoring may be used for reports and presentations. At each level the teacher must be able to identify the key characteristics of a quality report, such as a particular table, graph, diagram, or function. The teacher also will need to judge the quality of the student's reasoning, problem solving, and communication skills as displayed in the report or presentation. Holistic scoring by the teacher or by classmates could also be used to assess individual or group presentation of work.

An alternative to the holistic approach involves making separate judgments of the student's reasoning, problem solving, communication skills, and perhaps other categories that may be appropriate for a given report. Then, weighted in a manner that the teacher deems appropriate, these scores may be combined to determine a single grade for the report. Whether scoring in different categories is better than holistic scoring is debatable, but breaking down the grade into components probably makes the overall grade more understandable to the student and easier for the teacher to defend.

Specific ideas about evaluating extended projects are given in the teacher notes that accompany the extended project assignments.

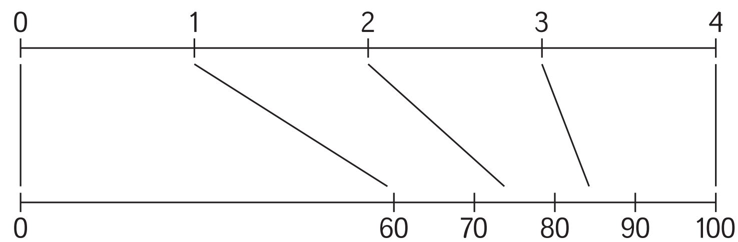
Assigning Grades

Since the *Core-Plus Mathematics* program provides a wide variety of assessment information, the teacher will be in a good position to assign appropriate grades. With such a wide choice of assessment opportunities, a word of caution is appropriate. It is easy to over-assess students, and care must be taken to avoid doing so. A quiz need not be given after every lesson nor an in-class exam after every unit. The developers believe it is best to vary assessment methods from lesson to lesson, and from unit to unit. If information on what students understand and are able to do is available from their homework and in-class work, it may not be necessary to take the time for a formal quiz after each lesson. Similarly, information from project work may replace an in-class exam. In some schools, teachers of each course work together to prepare and grade common assessments.

Deciding exactly how to weigh the various kinds of assessment information is a decision that you will need to make and communicate clearly to your students. Commonly, teachers assign a percentage of the final grade to different types of assessment such as in-class participation (dispositions), quizzes, exams, homework, take-home tasks, and projects. Sometimes teachers make more holistic judgments.

Rose Martin/CPMP

Some flexibility in grading is required. Recall that a score of 1 on a 0–4 scoring rubric indicates some understanding and that a score of 2 indicates a considerable amount of progress on challenging problem solving and reasoning assessment tasks. Even though 1/4 is 25% and 2/4 is 50%, which typically might be equated with failing grades, the level of understanding actually may be worthy of a grade of D or C. In short, the usual 90% for A, 80% for B, and so forth, may not work as a grading scale. It is possible to use a slightly different approach in which you directly translate a rubric score to the usual scale. Since 0%–59% is the interval of failure on this scale, there are really five intervals: 0–59, 60–69, 70–79, 80–89, and 90–100. In this approach, the teacher maps the five intervals of a rubric score directly into these intervals, using the following scheme or some variation on it:



With this translation, the teacher, students, and their parents simply (and quite legitimately) think of the rubric scores as equivalent to corresponding scores on the percent scale.

Whatever the method for determining grades, the developers recommend that you take advantage of the multiple sources of assessment information available, and do not base grades merely on quizzes and unit exams. The following is typical of the way the pilot- and field-test teachers weighted factors when assigning grades. Of course, individual teachers may decide on their own breakdowns.

Unit Tests	25%
Quizzes	25%
Homework	20%
Group Work / Participation	15%
Written / Oral Reports	10%
Notebooks / Journals	5%

One grading issue in using *Core-Plus Mathematics* relates to group work. Every student working in a group has a responsibility to help the group operate smoothly and to facilitate the learning to other group members. These outcomes are very important in this curriculum, and assessing and encouraging them for each student is no less important. For grading group assignments that result in a single report, all group members receive the same grade. So, how can individual grades be used to promote successful group work? One approach that seems to work well is to give extra credit to all group members when all members of a group do very well on a quiz or exam. Generally, this extra credit should be difficult, but not impossible, to earn.

Another grading issue arises from the fact that this curriculum is designed for all students. It intends to challenge the strongest students, while being accessible to all. Fair grading of such a wide range of students is not easy. Schools may choose to have two grading scales, "regular" or "core" and "honors" or "Core-Plus." The latter category typically requires consistently high-quality work on the core topics, plus work of similar quality on additional Extensions tasks in every unit. A grade in the Core-Plus category could be assigned more weight than a corresponding grade in the core category in the computation of a student's grade point average. For example, as many schools do with Advanced Placement classes, an A in the Core-Plus category could count as 5 quality points (or grade points), whereas an A in the core category might count as 4.

Tips on Student Work and Writing

Most teachers require students to maintain a three-ring binder containing their written work from investigations, Check Your Understanding, and On Your Own homework sets. Written assessments and projects that have been returned to students may be organized in the three-ring binder or held in a student portfolio that remains in the classroom at all times.

Writing Complete Responses

Helping students develop skills in writing complete and concise solutions is one of the goals of this curriculum. However, always writing thorough responses can unnecessarily slow student progress through the investigations. As a guideline, we suggest that during investigations, students should make notes of their thinking and discussion of ideas rather than use complete sentences. Investigation time can be thought of as draft work or getting ideas out for discussion. For investigation problems that ask students to explain reasoning or to compare, you may want to require complete-sentence responses. Student responses to the Summarize the Mathematics and Mathematics Toolkits entries should be more complete. If these responses are written following the class summary of important mathematical ideas, students will be able to write more thorough responses. Homework tasks from the On Your Own sets should also be thoroughly written.

Paperwork Management

In order to avoid being overwhelmed with paperwork and to avoid over-assessing students, teachers use a variety of sampling techniques. By selecting from the options below, individual teachers can develop their own scheme for paperwork management.

- Check only one paper per group for each investigation.
- Periodically check for completion of homework by a quick survey while students work on an investigation.
- Collect papers from the entire class, check the papers for completeness, and then examine one or two questions or parts of questions for correctness, complete-sentence responses, and thoroughness.
- Check different tasks for different students. (By assessing different items, teachers have a broader view of what students know and are able to do. Some teachers require students who wish to earn an A to do one or two Extensions tasks for every lesson.)

- Check for understanding by using a homework quiz. (Students copy their already-written responses to selected homework tasks such as Applications Task 2 Part b or Connections Task 4 Part f, or they simply circle their responses and hand in these items.)
- Collect notebooks and grade for completion at the end of each unit.
- Collect papers randomly from about one-third of the class for each assignment. Grade response to all or selected questions.
- Have students exchange papers in class as they go over the Connections tasks from the assignment. Students can address strengths and weaknesses of the responses and, with teacher guidance, assign a check, check-plus, or check-minus.
- If students are required to have all investigations and homework completed before taking a written assessment, it is helpful to keep a file card for each student that lists tasks yet to be completed.

Mathematics Toolkits

Some teachers also have students include a section (often containing colored paper) in the three-ring binder for the student-constructed Mathematics Toolkits. Other teachers require a separate spiral notebook, a booklet made from grid paper folded in half with a colored cover, or note cards on a ring. One advantage of a separate location for the Mathematics Toolkit is that students have long-term access to their compilations of class-generated definitions, examples, theorems, and mathematical concepts that they have developed throughout the year. If the Mathematics Toolkit is kept in the three-ring binder, it should be maintained throughout the year, even though other sections of the notebook may be emptied when starting each new unit. Students should also continue constructing their Mathematics Toolkits during all four courses of *Core-Plus Mathematics*. The toolkits give students a valuable reference, as well as a long-term record of their mathematical growth.

It is important to help students develop good work habits. Students' previous mathematics programs may not have emphasized writing. Students studying *Core-Plus Mathematics* for the first time will need to develop the ability to write clear, concise, mathematical explanations and justifications. Throughout the four courses, students will need to be encouraged to express their ideas completely and clearly. It is very important to have high expectations and hold students to high standards.

For over 20 years, with funding from the National Science Foundation, the Core-Plus Mathematics Project (CPMP) has been engaged in iterative cycles of research, design, and development; pilot testing in Michigan high schools, followed by revisions and refinement; and national field testing and further refinement prior to publication. The national field test sites involved 49 urban, suburban, and rural middle and high schools with diverse student populations in Alaska, California, Colorado, Georgia, Idaho, Iowa, Kentucky, Michigan, Missouri, Ohio, South Carolina, Texas, and Wisconsin.

Formative and summative evaluations have been a central feature of our development work. Individual school districts have also set into place trend studies of the effectiveness of *Core-Plus Mathematics*. In addition, we have been fortunate that mathematics education researchers have been attracted to our problem-based, inquiry-oriented approach to high school mathematics and have independently conducted and published efficacy research studies comparing the performance of CPMP students with comparable students using publisher-generated conventional high school mathematics programs (organized as Algebra I, Geometry, Advanced Algebra, and Precalculus).

Key Research Findings 1992–2012

Summarized below are key comparative evaluation and independent research findings reported during the last 20 years. Of particular note is the consistency of the findings in terms of students' conceptual understanding, reasoning, mathematical problem solving, college readiness, and dispositions toward mathematics.

Core-Plus Mathematics students:

- performed significantly better on tests of problem solving, applications, and conceptual understanding.
- elected to enroll in more high school mathematics courses.
- had positive attitudes and perceptions about mathematics.
- at the end of Course 3, performed significantly better on measures of conceptual understanding and problem solving in applied settings, but (using field-test materials) scored significantly lower than Algebra II students on a subtest of paper-and-pencil skills.
- performed as well on tests of paper-and-pencil algebraic skills (using published *Core-Plus Mathematics* texts).
- at the end of Course 3, performed at the level of the top-scoring country, the Netherlands, on a test composed of released 1995 TIMSS Twelfth-Grade Mathematical Literacy items.
- performed significantly better on the SAT Mathematics Test and as well on the ACT Mathematics Test.

- at the end of Course 4, outperformed comparable students on the calculus readiness portion of a mathematics placement test at a large Midwestern university. Of the 20 calculus readiness items, group means differed significantly on 12 of them, 11 in favor of CPMP students. The items were drawn from a bank of items available from the Mathematical Association of America.
- performed significantly better on a Test of Common Objectives in the areas of problem solving and reasoning and on a standardized test, ITED Mathematics: Concepts and Problem Solving, than did students who studied from publisher-produced, single-subject texts.

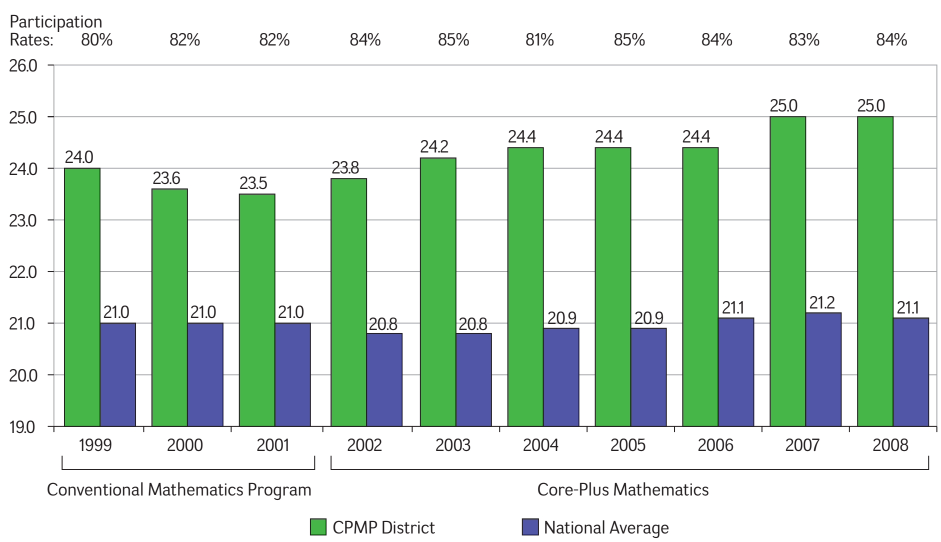
For individual abstracts of the evaluation and research studies on which these findings are based, see www.glencoe.com/glencoe_research/Math/cpmp.html.

College Readiness Indicators

Core-Plus Mathematics is an international-like four-year high school mathematics program intended to prepare students for college, careers, and daily life in contemporary society. In this section, several studies focusing on college readiness are highlighted.

ACT Test

Districts have monitored the ACT test performance of their students as they implemented CPMP and compared that performance to the performance of prior students in the district who had used a conventional mathematics program. Below is a portion of the trend data showing increasing participation rates and increasing average mathematics ACT scores for a suburban district in the Minneapolis, MN area.



College Placement Test

- CPMP students performed as well as comparable students from conventional programs on a college placement test at a large Midwestern university on basic algebra and advanced algebra subtests and performed significantly better on the calculus readiness portion of the test (Schoen & Hirsch, 2003).
- At the end of *Course 4: Preparation for Calculus*, CPMP students performed exceptionally well on the independently developed and research-based PCA Functions Test—a test related to understanding function concepts as a precursor to calculus. CPMP students outperformed comparable students on 21 of 25 questions (Engelke et al., 2005).

College Course Completions

- Results from a five-year longitudinal study showed that CPMP students after graduation from high school, completed first-year collegiate mathematics courses at about the same rate and with similar grades as all freshmen students with the same number of high school mathematics courses in two major research universities in two different states (Schoen, Ziebarth, Hirsch, & BrckaLorenz, 2010).
- The Minnesota Mathematics Achievement Project (MNMAP) researched the impact of curricula studied in high school (commercially developed, NSF-funded, or UCSMP) on the difficulty levels and grades of post-secondary students' mathematics courses. Most students in the MNMAP had studied the *Core-Plus Mathematics* program. When taking into account student background factors, no differences across high school curricula with respect to university mathematics grades or difficulty levels across eight semesters of college study were found. There also was no relationship between high school curricula and the number of college mathematics courses completed (Harwell et al., 2009; Post et al., 2010).

AP Calculus and AP Statistics

Trend data supplied by districts using *Core-Plus Mathematics* consistently show increased enrollments in AP Calculus and AP Statistics. Additionally, schools report that their students complete AP Calculus and AP Statistics at a higher rate and with a greater percentage of high scores on the AP examinations since the program was adopted. (www.wmich.edu/cpmp/schoolreports.html)

Performance Trends

In 2013, two comparison studies of *Core-Plus Mathematics* and more conventional curricula were completed by independent researchers at the University of Missouri–Columbia. The research was reported in the March and July 2013 issues of the prestigious *Journal for Research in Mathematics Education*. Abstracts of the two related studies follow.

Journal for Research in Mathematics Education

Curriculum and Implementation Effects on High School Students' Mathematics Learning From Curricula Representing Subject-Specific and Integrated Content Organizations

Douglas A. Grouws, James E. Tarr, Óscar Chávez, Ruthmae Sears, Victor M. Soria, and Rukiye D. Taylan
University of Missouri

This study examined the effect of 2 types of mathematics content organization on high school students' mathematics learning while taking account of curriculum implementation and student prior achievement. The study involved 2,161 students in 10 schools in 5 states. Within each school, approximately 1/2 of the students studied from an integrated curriculum [*Core-Plus Mathematics*] (Course 1) and 1/2 studied from a subject-specific curriculum [and publisher-generated: Glencoe/McGraw-Hill, McDougal Littel, Holt, Prentice Hall] (Algebra 1). Hierarchical linear modeling with 3 levels showed that students who studied from the integrated curriculum were significantly advantaged over students who studied from a subject-specific curriculum on 3 end-of-year outcome measures: Test of Common Objectives, Problem Solving and Reasoning Test, and a standardized achievement test. Opportunity to learn and teaching experience were significant moderating factors.

Journal for Research in Mathematics Education

The Effects of Content Organization and Curriculum Implementation on Students' Mathematics Learning in Second-Year High School Courses

James E. Tarr and Douglas A. Grouws
University of Missouri
Óscar Chávez
University of Texas at San Antonio
Victor M. Soria
University of Missouri

We examined curricular effectiveness in high schools that offered parallel paths in which students were free to study mathematics using 1 of 2 content organizational structures, an integrated approach [*Core-Plus Mathematics*] or a (traditional) subject-specific approach [and publisher-generated: Glencoe/McGraw-Hill, McDougal Littel, Holt, Prentice Hall]. The study involved 3,258 high school students, enrolled in either Course 2 or Geometry, in 11 schools in 5 geographically dispersed states. We constructed 3-level hierarchical linear models of scores on 3 end-of-year outcome measures: a test of common objectives, an assessment of problem solving and reasoning, and a standardized achievement test. Students in the integrated curriculum scored significantly higher than those in the subject-specific curriculum on the standardized achievement test. Significant student-level predictors included prior achievement, gender, and ethnicity. At the teacher level, in addition to Curriculum Type, the Opportunity to Learn and Classroom Learning Environment factors demonstrated significant power in predicting student scores, whereas Implementation Fidelity, Teacher Experience, and Professional Development were not significant predictors.

Content and Pedagogical Analyses of *Core-Plus Mathematics*

In October 1999, following an extensive content analysis and review of the evaluation studies completed to date, the U.S. Department of Education recognized *Core-Plus Mathematics* as one of six Exemplary School Mathematics Programs in the U.S.

A year later, AAAS reported findings of Project 2061's mathematics textbook evaluations related to high school algebra. The 12 textbooks evaluated included problem-based texts such as those from the Core-Plus Mathematics Project and the Interactive Mathematics Program (IMP), and also conventional, often more skill-oriented, Algebra 1 texts such as those from UCSMP (Scott Foresman), McDougal Littell, Glencoe/McGraw-Hill, and Prentice Hall.

This study evaluated each book's potential for teaching critical algebraic concepts, such as representing variable quantities and modeling with functions, and analyzed how well the content was developed through instructional strategies that are consistent with research on how students learn. Of the 12 texts evaluated, 7 were identified as having "potential for helping students learn algebra." Of these 7 texts, *Core-Plus Mathematics: Contemporary Mathematics in Context* received the *highest* rating.

Further detail on this textbook evaluation study can be found at www.project2061.org.

Meta-Evaluation Studies of *Core-Plus Mathematics*

Two meta-evaluations of the research on the efficacy of 1st edition *Core-Plus Mathematics* have been conducted. Although each used somewhat different criteria, the direction of the reported findings is in general agreement.

The corpus of evaluation studies of *Core-Plus Mathematics* was included in an exhaustive, "best evidence" review of hundreds of published and unpublished papers by Slavin, Lake, and Groff (2008). The review had stringent inclusion criteria for classroom studies, including only those studies that met the following criteria:

- Schools or classrooms using each program had to be compared to randomly assigned or well-matched control groups.
- Study duration had to be at least 12 weeks.
- Outcome measures had to be assessments of the mathematics being taught in all classes. Almost all are standardized tests or state assessments.
- The review placed particular emphasis on studies in which schools, teachers, or students were assigned at random to experimental or control groups.

The *Core-Plus Mathematics* program was one of just two *Standards*-based curricula placed in the "Limited Evidence of Effectiveness" category. All other high school mathematics programs for which evidence was reviewed were in lower categories such as "Insufficient Evidence" or "No Qualifying Studies" (Slavin et al., 2008).

Finally, in June 2009, following an extensive review of research on education-related programs by the American Institute for Research and Strategic Ed Solutions for the Business-Higher Education Forum, the *Core-Plus Mathematics* program was recognized as one of 35 programs (across all subject areas) in the U.S. that increase student achievement and improve college readiness.

Selected References

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As you teach courses using *Core-Plus Mathematics* materials, you may find some of the following resources helpful.

Professional Development

Contact your McGraw-Hill Education sales representative for information on professional development options for the *Core-Plus Mathematics* program. For specific sales representative information, please contact the MHE service center.

800-334-7344 **Fax: 614-860-1877**

Ordering: MMH_OrderServices@mheducation.com

General Inquiries: SEG_CustomerService@mheducation.com

McGraw-Hill School Education
PO Box 182605
Columbus, Ohio 43218

Books, Articles, and Online Resources

Barton, M. L., & Heidema, C. (2002). *Teaching reading in mathematics: A supplement* (2nd ed.). Alexandria, VA: ASCD.

Cohen, E. G. (1999). Complex instruction: equity in cooperative learning classrooms. *Theory into Practice, 38*(2) 80–86.

Davidson, N. (Ed.). (1990). *Cooperative learning in mathematics: A handbook for teachers*. Menlo Park, CA: Addison Wesley.

Herrel, A. & Jordan, M. (2012). *Fifty strategies for teaching english language learners (4th ed.)*. Boston, MA: Pearson Publication.

Illustrative Mathematics (www.illustrativemathematics.org) provides guidance to states, assessment consortia, testing companies, and curriculum developers by illustrating the range and types of mathematical work that students experience in a faithful implementation of the *Common Core State Standards*, and by publishing other tools that support implementation of the standards.

Inside Mathematics (www.insidemathematics.org) is a professional resource for educators passionate about improving students' mathematics learning and performance. The site includes tools for observation and reflection by teachers, coaches, and administrators intent on improving student learning.

Lenses on Learning modules for leadership teams is available at www2.edc.org/cdt/cdt/cdt_lol1.html.

The Mathematics Assessment Project (MAP; www.mathshell.org/ba_mars.htm) is developing formative assessment lessons and rich summative performance tasks to support the *Common Core State Standards* emphasizing the vital mathematical practices they require. These tasks might be used as formative evaluation of your mathematics program.