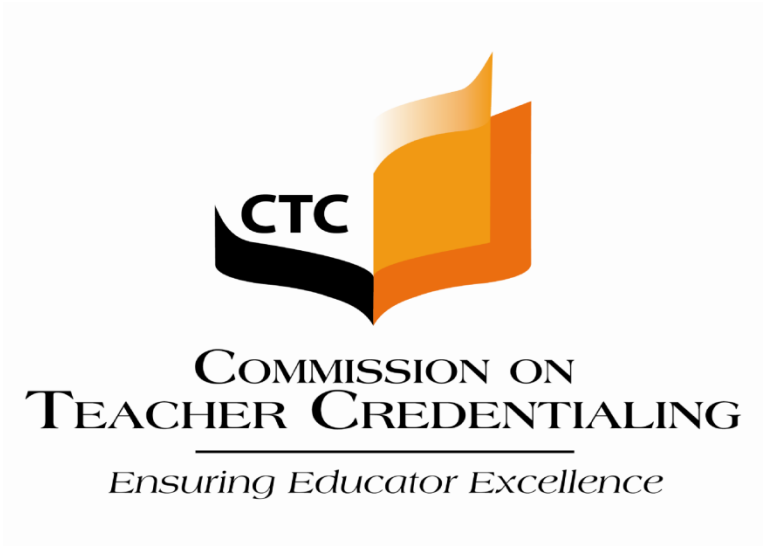
Alignment Matrix of Traditional Mathematics and Foundational-Level Mathematics Subject Matter Requirements and Program Elements



Subject Matter Requirements for California's Common Core State Standards

Updated September 2013

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Alignment Matrix of Mathematics and Foundational-Level Mathematics Subject Matter Requirements (SMRs) and Program Elements (2013)

This matrix provides a structure through which prospective program sponsors can identify and link program elements to each sub-domain. Program elements are documents that demonstrate how the sponsor will ensure that candidates have multiple opportunities to learn and utilize the subject matter requirements for Mathematics and Foundational Level Mathematics (FLM) teachers. Examples of documents to identify include course syllabi, specific lectures or assignments, and assessments.

Subject matter requirements (SMRs) for Foundational Level Mathematics are identified by italics, and by an open cell under the Foundational Level Mathematics heading. The letters for the FLM SMRs are shown in parentheses ( ).

For Initial Program Proposals, incorporate the syllabi into the proposal so that reviewers can cross-reference coursework identified in the matrix.

For Approved Program Sponsors when SMRs change, enter the course number, a description of the lecture, assignment, or assessment. DO NOT submit the syllabi. In addition, please submit a list of required mathematics courses for the subject matter preparation program, including course title, number, and short description.

Example:

Subject Matter Requirements	**Coursework, Assignments, Assessments, etc.**	
	FOUNDATIONAL LEVEL MATHEMATICS	**MATHEMATICS**
Domain 1: Algebra Candidates demonstrate an understanding of the foundations of algebra as outlined in California's Common Core Content Standards for Mathematics (Grade 7, Grade 8, and High School). Candidates demonstrate a depth and breadth of conceptual knowledge to ensure a rigorous view of algebra and its underlying structures. They are skilled at symbolic reasoning and use algebraic skills and concepts to model a variety of problem-solving situations. They understand the power of mathematical abstraction and symbolism.		
1.1 Algebraic Structures a. Demonstrate knowledge of why the real and complex numbers are each a field, and that particular rings are not fields (e.g., integers, polynomial rings, matrix rings)		**Math course XXX, example from lecture, Description of an assignment** **Math course XXY, title of lecture, brief description of project**
b. (a) Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if a < b and c < 0, then ac > bc)	**Math course XXE, lecture topic, description of an assignment, test item**	**Math course XXE, lecture topic, description of an assignment, description of test item**

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Subject Matter Requirements	Coursework, Assignments, Assessments, etc.	
	FOUNDATIONAL LEVEL MATHEMATICS	**MATHEMATICS**
Domain 1: Algebra Candidates demonstrate an understanding of the foundations of algebra as outlined in California's Common Core Content Standards for Mathematics (Grade 7, Grade 8, and High School). Candidates demonstrate a depth and breadth of conceptual knowledge to ensure a rigorous view of algebra and its underlying structures. They are skilled at symbolic reasoning and use algebraic skills and concepts to model a variety of problem-solving situations. They understand the power of mathematical abstraction and symbolism.		
1.2 Algebraic Structures *a.* Demonstrate knowledge of why the real and complex numbers are each a field, and that particular rings are not fields (e.g., integers, polynomial rings, matrix rings)		
c. (a) Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if a < b and c < 0, then ac > bc)		
d. (b) Demonstrate knowledge that the rational numbers and real numbers can be ordered and that the complex numbers cannot be ordered, but that any polynomial equation with real coefficients can be solved in the complex field		
e. (c) Identify and translate between equivalent forms of algebraic expressions and equations using a variety of techniques (e.g., factoring, applying properties of operations)		
f. (d) Justify the steps in manipulating algebraic expressions and solving algebraic equations and inequalities		
g. (e) Represent situations and solve problems using algebraic equations and inequalities		

Subject Matter Requirements	Coursework, Assignments, Assessments, etc.	
	FOUNDATIONAL LEVEL MATHEMATICS	**MATHEMATICS**
1.3 Polynomial Equations and Inequalities *a. Analyze and solve polynomial equations with real coefficients using:* ♦ the Fundamental Theorem of Algebra ♦ *the Rational Root Theorem for polynomials with integer coefficients* ♦ the Conjugate Root Theorem for polynomial equations with real coefficients ♦ the Binomial Theorem		
b. Prove and use the Factor Theorem and the quadratic formula for real and complex quadratic polynomials		
c. Solve polynomial inequalities		
1.3 Functions *a. Analyze general properties of functions (i.e., domain and range, one-to-one, onto, inverses, composition, and differences between relations and functions) and apply arithmetic operations on functions*		
b. Analyze properties of linear functions (e.g., slope, intercepts) using a variety of representations		
c. Demonstrate knowledge of why graphs of linear inequalities are half planes and be able to apply this fact		
d. Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems)		
e. Analyze properties of exponential and logarithmic functions in a variety of ways (e.g., graphing, solving problems)		

Subject Matter Requirements	Coursework, Assignments, Assessments, etc.	
	FOUNDATIONAL LEVEL MATHEMATICS	**MATHEMATICS**
f. Model and solve problems using nonlinear functions		
1.4 Linear Algebra a. Understand and apply the geometric interpretation and basic operations of vectors in two and three dimensions, including their scalar multiples		
b. Prove the basic properties of vectors (e.g., perpendicular vectors have zero dot product)		
c. Understand and apply the basic properties and operations of matrices and determinants (e.g., to determine the solvability of linear systems of equations)		
d. (a) Analyze the properties of proportional relationships, lines, linear equations, and their graphs, and the connections between them		
e. (b) Model and solve problems using linear equations, pairs of simultaneous linear equations, and their graphs		
Domain 2: Geometry Candidates demonstrate an understanding of the foundations of geometry outlined in California's Common Core Content Standards for Mathematics (Grade 7, Grade 8, and High School). Candidates demonstrate a depth and breadth of conceptual knowledge to ensure a rigorous view of geometry and its underlying structures. They demonstrate an understanding of axiomatic systems and different forms of logical arguments. Candidates understand, apply, and prove theorems relating to a variety of topics in two and three-dimensional geometry, including coordinate, synthetic, non-Euclidean, and transformational geometry.		
2.1 Plane Euclidean Geometry *a. Apply the Parallel Postulate and its implications and justify its equivalents (e.g., the Alternate Interior Angle Theorem, the angle sum of every triangle is 180 degrees)*		
b. Demonstrate knowledge of complementary, supplementary, and vertical angles		

Subject Matter Requirements	Coursework, Assignments, Assessments, etc.	
	FOUNDATIONAL LEVEL MATHEMATICS	**MATHEMATICS**
c. Prove theorems, justify steps, and solve problems involving similarity and congruence		
d. Apply and justify properties of triangles (e.g., the Exterior Angle Theorem, concurrence theorems, trigonometric ratios, triangle inequality, Law of Sines, Law of Cosines, the Pythagorean Theorem and its converse)		
e. Apply and justify properties of polygons and circles from an advanced standpoint (e.g., derive the area formulas for regular polygons and circles from the area of a triangle)		
f. Identify and justify the classical constructions (e.g., angle bisector, perpendicular bisector, replicating shapes, regular polygons with 3, 4, 5, 6, and 8 sides)		
2.2 Coordinate Geometry *a. Use techniques in coordinate geometry to prove geometric theorems*		
b. Model and solve mathematical and real-world problems by applying geometric concepts to two-dimensional figures		
c. Translate between the geometric description and the equation for a conic section		
d. Translate between rectangular and polar coordinates and apply polar coordinates and vectors in the plane		
2.3 Three-Dimensional Geometry *a. Demonstrate knowledge of the relationships between lines and planes in three dimensions (e.g., parallel, perpendicular, skew, coplanar lines)*		

Subject Matter Requirements	Coursework, Assignments, Assessments, etc.	
	FOUNDATIONAL LEVEL MATHEMATICS	**MATHEMATICS**
b. Apply and justify properties of three-dimensional objects (e.g., the volume and surface area formulas for prisms, pyramids, cones, cylinders, spheres)		
c. Model and solve mathematical and real-world problems by applying geometric concepts to three-dimensional figures		
2.4 Transformational Geometry *a. Demonstrate knowledge of isometries in two- and three-dimensional space (e.g., rotation, translation, reflection), including their basic properties in relation to congruence*		
b. Demonstrate knowledge of dilations (e.g., similarity transformations or change in scale factor), including their basic properties in relation to similarity, volume, and area		
Domain 3: Number and Quantity Candidates demonstrate an understanding of number theory and a command of number sense as outlined in California's Common Core Content Standards for Mathematics (Grade 6, Grade 7, Grade 8, and High School). Candidates demonstrate a depth and breadth of conceptual knowledge to ensure a rigorous view of number theory and its underlying structures. They prove and use properties of natural numbers. They formulate conjectures about the natural numbers using inductive reasoning, and verify conjectures with proofs.		
3.1 The Real and Complex Number Systems *a. Demonstrate knowledge of the properties of the real number system and of its subsets*		
b. Perform operations and recognize equivalent expressions using various representations of real numbers (e.g., fractions, decimals, exponents)		
c. Solve real-world and mathematical problems using numerical and algebraic expressions and equations		
d. Apply proportional relationships to model and solve real-world and mathematical problems		

Subject Matter Requirements	Coursework, Assignments, Assessments, etc.	
	FOUNDATIONAL LEVEL MATHEMATICS	**MATHEMATICS**
e. Reason quantitatively and use units to solve problems (i.e., dimensional analysis)		
f. Perform operations on complex numbers and represent complex numbers and their operations on the complex plane		
3.2 Number Theory *a. Prove and use basic properties of natural numbers (e.g., properties of divisibility)*		
b. Use the principle of mathematical induction to prove results in number theory		
c. Apply the Euclidean Algorithm		
d. Apply the Fundamental Theorem of Arithmetic (e.g., find the greatest common factor and the least common multiple; show that every fraction is equivalent to a unique fraction where the numerator and denominator are relatively prime; prove that the square root of any number, not a perfect square number, is irrational)		
Domain 4: Probability and Statistics Candidates demonstrate an understanding of statistics and probability distributions in California's Common Core Content Standards for Mathematics (Grade 7, Grade 8, and High School). Candidates demonstrate a depth and breadth of conceptual knowledge to ensure a rigorous view of probability and statistics and their underlying structures. They solve problems and make inferences using statistics and probability distributions.		
4.1 Probability *a. Prove and apply basic principles of permutations and combinations*		
b. *Illustrate finite probability using a variety of examples and models (e.g., the fundamental counting principles, sample space)*		
c. Use and explain the concepts of conditional probability and independence		

Subject Matter Requirements	Coursework, Assignments, Assessments, etc.	
	FOUNDATIONAL LEVEL MATHEMATICS	**MATHEMATICS**
d. Compute and interpret the probability of an outcome, including the probabilities of compound events in a uniform probability model		
e. Use normal, binomial, and exponential distributions to solve and interpret probability problems		
f. Calculate expected values and use them to solve problems and evaluate outcomes of decisions		
4.2 Statistics *a. Compute and interpret the mean and median of both discrete and continuous distributions*		
b. Compute and interpret quartiles, range, interquartile range, and standard deviation of both discrete and continuous distributions		
c. Select and evaluate sampling methods appropriate to a task (e.g., random, systematic, cluster, convenience sampling) and display the results		
d. Apply the method of least squares to linear regression		
e. Apply the chi-square test		
f. (d) Interpret scatter plots for bivariate data to investigate patterns of association between two quantities (e.g., correlation), including the use of linear models		
g. (e) Interpret data on a single count or measurement variable presented in a variety of formats (e.g., dot plots, histograms, box plots)		
h. Demonstrate knowledge of P-values and hypothesis testing		

Subject Matter Requirements	Coursework, Assignments, Assessments, etc.	
	FOUNDATIONAL LEVEL MATHEMATICS	**MATHEMATICS**
i. Demonstrate knowledge of confidence intervals		
Domain 5: Calculus Candidates demonstrate an understanding of trigonometry and calculus as outlined in California's Common Core Content Standards for Mathematics (High School). Candidates demonstrate a depth and breadth of conceptual knowledge to ensure a rigorous view of trigonometry and calculus and their underlying structures. They apply the concepts of trigonometry and calculus to solving problems in real-world situations.		
5.1 Trigonometry *a. Prove that the Pythagorean Theorem is equivalent to the trigonometric identity $sin^2x + cos^2x = 1$ and that this identity leads to $1 + tan^2x = sec^2x$ and $1 + cot^2x = csc^2x$*		
b. Prove and apply the sine, cosine, and tangent sum formulas for all real values		
c. (b) Analyze properties of trigonometric functions in a variety of ways (e.g., graphing and solving problems, using the unit circle)		
d. (c) Apply the definitions and properties of inverse trigonometric functions (i.e., arcsin, arccos, and arctan)		
e. Apply polar representations of complex numbers (e.g., DeMoivre's Theorem)		
f. (d) Model periodic phenomena with periodic functions		
g. (e) Recognize equivalent identities, including applications of the half-angle and double-angle formulas for sines and cosines		
5.2 Limits and Continuity a. Derive basic properties of limits and continuity, including the Sum, Difference, Product, Constant Multiple, and Quotient Rules, using the formal definition of a limit		
b. Show that a polynomial function is continuous at a point		

Subject Matter Requirements	Coursework, Assignments, Assessments, etc.	
	FOUNDATIONAL LEVEL MATHEMATICS	**MATHEMATICS**
c. Apply the intermediate value theorem, using the geometric implications of continuity		
5.3 Derivatives and Applications a. Derive the rules of differentiation for polynomial, trigonometric, and logarithmic functions using the formal definition of derivative		
b. Interpret the concept of derivative geometrically, numerically, and analytically (i.e., slope of the tangent, limit of difference quotients, extrema, Newton's method, and instantaneous rate of change)		
c. Interpret both continuous and differentiable functions geometrically and analytically and apply Rolle's theorem, the mean value theorem, and L'Hôpital's rule		
d. Use the derivative to solve rectilinear motion, related rate, and optimization problems		
e. Use the derivative to analyze functions and planar curves (e.g., maxima, minima, inflection points, concavity)		
f. Solve separable first-order differential equations and apply them to growth and decay problems		
5.4 Integrals and Applications a. Derive definite integrals of standard algebraic functions using the formal definition of integral		
b. Interpret the concept of a definite integral geometrically, numerically, and analytically (e.g., limit of Riemann sums)		
c. Prove the fundamental theorem of calculus, and use it to interpret definite integrals as antiderivatives		

Subject Matter Requirements	Coursework, Assignments, Assessments, etc.	
	FOUNDATIONAL LEVEL MATHEMATICS	**MATHEMATICS**
d. Apply the concept of integrals to compute the length of curves and the areas and volumes of geometric figures		
5.5 Sequences and Series *a. Derive and apply the formulas for the sums of finite arithmetic series and finite and infinite geometric series (e.g., express repeating decimals as a rational number)*		
b. Determine convergence of a given sequence or series using standard techniques (e.g., ratio, comparison, integral tests)		
c. Calculate Taylor series and Taylor polynomials of basic functions		