Math 270 Basic Discrete Math
Practice Test 3
Sections 5.1, 5.2, 5.3, 5.4, 5.6, 5.7

Name: (Please Print) ____

Directions: Answer the problems below. You may use scientific (non-graphing) calculators, but no other electronic devices. Show all work.

1. Prove, using mathematical induction, that for all integers $n \geq 1$,

$$3 + 7 + 11 + \cdots + (4n - 1) = 2n(n + 1) - n.$$

2. Let $a_1, a_2, a_3, \ldots$ be the sequence defined recursively as follows:

$$a_1 = 1,\ a_2 = 20, \text{ and for all } k \geq 3,\ a_k = 5a_{k-1} + 6a_{k-2}.$$

Use strong induction to prove that for all integers $n \geq 1$, $a_n \leq 6^n$.

3. Provide short responses for parts a.-d. below.

a. Calculate each of the following:

i. $\prod_{i=1}^{4}(2i) =$

ii. $\sum_{i=1}^{4}(2i-1) =$

iii. $\frac{4!}{2!} =$

iv. $\binom{6}{2} =$

v. $\binom{6}{4} =$

vi. $\binom{6}{0} =$

b. Suppose the sequence $a_1, a_2, a_3, \ldots$ begins with the terms $8, -27, 64, -125, 216, \ldots$. Find an explicit formula for a_n.

c. Write the product $(1-t)(1-2t^2)(1-3t^3)(1-4t^4)$ using product notation.

d. Transform the sum $\sum_{j=3}^{n+1} \frac{j^2-1}{n-j+2}$ by making the change of variable $i = j - 2$.

4. Find explicit formulas for the following recurrence relations. (You do *not* need to prove your answers are correct.) Simplify your answers as much as possible: for full credit your answers should include no summation or product notation.

a. $a_1 = 1$, $a_k = a_{k-1} + 2$ for all $k \geq 2$.

b. $b_1 = 2$, $b_k = k \cdot b_{k-1}$ for all $k \geq 2$.

c. $c_1 = 0$, $c_k = c_{k-1} + 2k$ for all $k \geq 2$.

d. $d_1 = 1$, $d_k = 2d_{k-1} + 1$ for all $k \geq 2$.