# Math 270 Basic Discrete Math
## Practice Test 4
Sections 5.8, 5.9, 6.1, 8.1, 8.2, 8.3

Name: (Please Print) _____

**Directions:** Answer the problems below. You may use scientific (non-graphing) calculators, but no other electronic devices. Show all work.

**1.** Let $a_1, a_2, a_3, \ldots$ be the sequence defined recursively as follows:

$$a_1 = 9, \; a_2 = 21, \text{ and for all } k \geq 3, \; a_k = 5a_{k-1} - 6a_{k-2}.$$

Find an explicit formula for $a_n$.

**2.** In this problem, let $A = \{1, 2, 3, 4, 5, 6\}$. Answer a.-c. below.

**a.** Describe a relation $R$ on $A$ which is reflexive but is not symmetric by (i) drawing the digraph for $R$ and (ii) listing the elements of $R$ in set-roster notation.

**b.** Draw the digraph for an equivalence relation $S$ on $A$ which has three distinct equivalence classes: $\{1, 2\}$, $\{3\}$, and $\{4, 5, 6\}$. (You only need to draw the digraph.)

**c.** Let $T$ be the equivalence relation on $A$ given by

$$x \, T \, y \quad \Leftrightarrow \quad 3 | (x^2 - y^2).$$

What are $T$'s distinct equivalence classes?

**3.** Answer parts a. and b. below.

**a.** Let the universe $\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and let

$$A = \{1, 2, 3, 4\}, \quad B = \{2, 4, 6, 8, 10\}, \quad \text{and} \quad C = \{3, 6, 9\}.$$

Calculate each of the following:

**i.** $A \cup B =$

**ii.** $B \cap C =$

**iii.** $(A \cup B) - C =$

**iv.** $A \cap B \cap C =$

**v.** $(A \cup B)^c =$

**b.** Draw a Venn diagram for three sets $A, B, C$ which satisfy the following conditions:

$$A \subseteq B, \ A \cap C \neq \varnothing, \ B \cap C \neq \varnothing.$$

**4.** Define a set $S$ of integers recursively as follows:

   I. Base: $3 \in S$.

  II. Recursion: if $k \in S$, then

        II(a) $k + 6 \in S$

  III. Restriction: Nothing is in $S$ other than objects defined in $I, II$ above.

Use structural induction to prove that every integer $n \in S$ is divisible by 3.

**5**. Answer parts a.-c. below.

**a.** Define the sets

$$A = \{n \in \mathbb{Z} \mid n = 4a + 1 \text{ for some } a \in \mathbb{Z}\}, \text{ and}$$
$$B = \{m \in \mathbb{Z} \mid m = 4b + 3 \text{ for some } b \in \mathbb{Z}\}.$$

Are $A$ and $B$ disjoint? Why or why not?

**b.** Express the power set $\mathscr{P}(\{1, 2, 3\})$ in set-roster notation.