

Math 270 Basic Discrete Math
Practice Test 5
Sections 9.2, 9.1, 9.3, 9.5, 9.6, 10.4

Name: (Please Print) _____

Directions: Answer the problems below. You may use scientific (non-graphing) calculators, but no other electronic devices. Show all work.

1. The code to a particular combination lock consists of an ordered selection of five numbers, each from 1 through 100.

a. How many different codes are possible?

b. How many codes have no repeated number?

c. How many codes have the same first and last number?

2. In the Commonwealth of Pennsylvania, car license plates consist of three upper-case letters (from A-Z) followed by four numerals (0-9), in the form $LLL-NNNN$. Order matters: $ABC-1234$ is not the same as $BAC-4321$.

Answer parts a.-d. below. Your answer may include products, division, exponents, and factorials (such as, say, $2^2 \cdot 3 \cdot 4!$) if needed; you do not need to simplify.

a. How many different license plates does Pennsylvania have available?

b. How many license plates have no repeated numerals? (Repeated letters are allowed.)

c. How many license plates have no repeated letters? (Repeated numerals are allowed.)

d. What is the probability that a randomly chosen license plate has neither a repeated letter nor numeral?

3. Provide short answers for parts a.-d. below.

a. How many different 4-permutations are there of the elements of the set $\{a, b, c, d, e, f, g\}$?

b. How many solutions are there to the equation $x+y+z = 10$ in nonnegative integers x, y, z ?

c. Suppose a graph G has six vertices with degrees 1, 1, 3, 4, 4, 5. Can G be a tree?

d. In how many distinguishable ways can the letters of the word *COMBINATORICS* be arranged in order?

4. Your instructor is designing an exam for Math 270. The exam will consist of five problems chosen from a collection of 15: five of these problems are 'easy', six problems are 'average', and four problems are 'hard'. Answer the problems below but you do not need to simplify: your answers may include products, division, sums, differences, exponents, factorials, and binomial coefficients (i.e. $\binom{n}{k}$) if appropriate.

a. How many possible exams can be constructed this way?

b. How many of these exams have exactly one hard problem?

c. How many of these exams consist solely of average problems?

d. Suppose your instructor has written two problems that are very, very similar: how many exams do not contain both of these problems (but may contain one)?

5. Answer parts a. and b. below.

a. Draw a graph G on 7 vertices with exactly 6 edges that is not a tree, and clearly state why it is not a tree.

b. Suppose T is a tree with exactly 5 internal vertices, of degrees 2, 3, 3, 4, 5; all other vertices of T are leaves. How many leaves does T have?