

Math 270 - Basic Discrete Mathematics
Practice Quiz on Section 4.1

Solutions

Directions: Answer the problems given below.

1. Assume that m and n are particular integers.

a. Is $4m + 7$ odd? Why or why not?

$$\begin{aligned} \text{Yes: } 4m + 7 &= 4m + 6 + 1 \\ &= 2(2m + 3) + 1, \\ &\text{and } 2m + 3 \text{ is an integer by closure.} \end{aligned}$$

b. Is $8m - 10n$ even? Why or why not?

$$\begin{aligned} \text{Yes: } 8m - 10n &= 2(4m - 5n), \\ &\text{and } 4m - 5n \text{ is an integer by closure.} \end{aligned}$$

c. Assuming that $m > 1$, is $m^2 + 4m + 4$ composite? Why or why not?

$$\begin{aligned} \text{Yes. } m^2 + 4m + 4 &= (m + 2)^2 = (m + 2)(m + 2). \\ \text{Since } m > 1, m + 2 > 1, \text{ so } (m + 2)^2 > (m + 2). \\ \text{Letting } a &= (m + 2)^2, r = m + 2, s = m + 2, \text{ we have} \\ a &= rs, 1 < r < a, 1 < s < a, \text{ so } a \text{ is composite.} \end{aligned}$$

2. Prove the following statement: There exist real numbers x and y such that

$$\sqrt{x + y} = \sqrt{x} + \sqrt{y}.$$

Proof: Let $x = 0, y = 0$. Since $0 \in \mathbb{R}$,
 x and y are real, and

$$\sqrt{x + y} = \sqrt{0 + 0} = \sqrt{0} = 0 = \sqrt{0} + \sqrt{0} = \sqrt{x} + \sqrt{y}. \quad \square$$