

Math 270 - Basic Discrete Mathematics
Practice Quiz on Section 4.3

Solution

Directions: Answer the problems given below.

1. The following numbers are all rational: write each as a fraction of integers.

a. $17 = \frac{17}{1}$

b. $-6.25 = \frac{-25}{4}$

c. $0.2828282828\dots = \frac{28}{99}$

$x = 0.282828\dots$: $10x = 28.282828\dots$
 $10x - x = 28 \rightarrow 99x = 28$, so $x = \frac{28}{99}$.

2. Prove that for all rational numbers r and s , the number $2r - 3s$ is rational.

Proof: Let r, s be arbitrary rational numbers. Then $\exists a, b, c, d \in \mathbb{Z}$

such that $r = \frac{a}{b}$, $s = \frac{c}{d}$, $b \neq 0$, $d \neq 0$.

Then

$$2r - 3s = 2\left(\frac{a}{b}\right) - 3\left(\frac{c}{d}\right) = \frac{2ad - 3bc}{bd}$$

Since $a, b, c, d \in \mathbb{Z}$, both $2ad - 3bc \in \mathbb{Z}$ and $bd \in \mathbb{Z}$ by closure.

Since $b \neq 0$ and $d \neq 0$, $bd \neq 0$ by the Zero Product Property (ZPP).

Therefore, $2r - 3s$ is rational by definition. Since r, s were arbitrary, the proof holds. \Rightarrow