Math 270 - Basic Discrete Mathematics
Practice Quiz on Section 4.7
Solutions

Directions: Answer the problem given below.

1. Prove that for all integers m and n, if mn is even then m is even or n is even.

Proof: We prove this by contraposition: the contrapositive is "For all integers m and n, if m is odd and n is odd then mn is odd."

Let $m, n \in \mathbb{Z}$ be arbitrary and suppose both are odd.

Then $\exists k, \ell \in \mathbb{Z}$ such that $m = 2k + 1$ and $n = 2\ell + 1$.

So,

$$mn = (2k+1)(2\ell+1) = 4k\ell + 4k + 4\ell + 1$$
$$= 2(2k\ell + 2k + 2\ell) + 1.$$

Since $2k\ell + 2k + 2\ell \in \mathbb{Z}$ by closure, it follows that mn is odd (by definition), and the proof now follows as m, n were arbitrary. ∎