

Math 270 - Basic Discrete Mathematics
Practice Quiz on Section 4.8

Solutions

Directions: Answer the problems given below.

1. Is the number $5 + 3\sqrt{2}$ rational or irrational? Why or why not?

$5 + 3\sqrt{2}$ is irrational: if it were rational, say

$$5 + 3\sqrt{2} = \frac{a}{b} \text{ with } a, b \in \mathbb{Z}, b \neq 0, \text{ then}$$

$$\sqrt{2} = \frac{a - 5b}{3b}, \text{ which is rational by closure and the ZPP,} \\ \text{a contradiction as } \sqrt{2} \text{ is irrational!}$$

2. Is it true that whenever x, y are positive irrational numbers, $x + y$ is also irrational? Why or why not?

No, it is not true: let $x = \sqrt{2}$ and

$$y = 10 - \sqrt{2}. \text{ Observe that } x > 0, y > 0$$

(since $\sqrt{2} \approx 1.4 < 10$) and x is irrational.

$y = 10 - \sqrt{2}$ is also irrational, by the same

reasoning as in Problem 1.

And since $x + y = 10 = \frac{10}{1} \in \mathbb{Q}$, we see that

we have a counterexample to the given statement.