Math 270 - Basic Discrete Mathematics
Practice Quiz on Section 5.2

Solutions

Directions: Answer the problem given below.

1. Prove using mathematical induction that for every integer $n \geq 1$,

$$\frac{2}{1\cdot 2} + \frac{2}{2\cdot 3} + \frac{2}{3\cdot 4} + \cdots + \frac{2}{n(n+1)} = \frac{2n}{n+1}.$$

Proof: Let $P(n) =$ " $\frac{2}{1.2} + \frac{2}{2.3} + \cdots + \frac{2}{n(n+1)} = \frac{2n}{n+1}$. ".

We prove $P(n)$ holds for all $n \geq 1$ by induction on n.

Base case ($n=1$): the LHS of $P(1)$ is $\frac{2}{1\cdot 2} = 1$,

and the RHS is $\frac{2\cdot 1}{1+1} = 1$, so LHS = RHS, hence $P(1)$ holds.

Inductive Step: Let $k \geq 1$ be an arbitrary integer and suppose $P(k)$ holds, so

$$\frac{2}{1\cdot 2} + \frac{2}{2\cdot 3} + \cdots + \frac{2}{k(k+1)} = \frac{2k}{k+1}.$$

We must show that $P(k+1)$ holds, i.e. that

$$\frac{2}{1\cdot 2} + \frac{2}{2\cdot 3} + \cdots + \frac{2}{(k+1)(k+2)} = \frac{2(k+1)}{k+2}.$$

Starting with the LHS of $P(k+1)$,

$$\frac{2}{1\cdot 2} + \frac{2}{2\cdot 3} + \cdots + \frac{2}{(k+1)(k+2)} = \left(\frac{2}{1\cdot 2} + \cdots + \frac{2}{k(k+1)}\right) + \frac{2}{(k+1)(k+2)}$$

$$= \frac{2k}{k+1} + \frac{2}{(k+1)(k+2)} \qquad \text{(by the I.H.)}$$

$$= \frac{2k(k+2)+2}{(k+1)(k+2)} = \frac{2(k^2+2k+1)}{(k+1)(k+2)} = \frac{2(k+1)}{k+2},$$

hence $P(k+1)$ is true. This proves the inductive step, and since the base case holds, $P(n)$ is true for all integers $n \geq 1$ by the PMI. ∎