Math 270 - Basic Discrete Mathematics
Practice Quiz on Section 5.3
Solutions

Directions: Answer the problem given below.

Same as $3 \mid (7^k - 4^k)$

1. Prove using mathematical induction that for any integer $n \geq 1$, $7^n - 4^n$ is divisible by 3.

Proof: Let $P(n) =$ "$3 \mid (7^n - 4^n)$": we prove $P(n)$ holds for all $n \geq 1$ by induction on n.

Base Case ($n=1$): When $n = 1$, $7^n - 4^n = 7 - 4 = 3$, and clearly $3 \mid 3$, so $P(1)$ holds.

Induction Step: Let $k \geq 1$ be arbitrary and suppose $P(k)$ holds, i.e. $3 \mid (7^k - 4^k)$. We must show $P(k+1)$ holds, i.e., that $3 \mid (7^{k+1} - 4^{k+1})$.

Observe that

$$
\begin{aligned}
7^{k+1} - 4^{k+1} &= 7 \cdot 7^k - 4 \cdot 4^k \\
&= 3 \cdot 7^k + 4 \cdot 7^k - 4 \cdot 4^k \\
&= 3 \cdot 7^k + 4(7^k - 4^k).
\end{aligned}
$$

Clearly $3 \mid (3 \cdot 7^k)$, and it follows that $3 \mid (4(7^k - 4^k))$ from the I.H., so 3 must divide their sum. That is, $3 \mid (7^{k+1} - 4^{k+1})$, so $P(k+1)$ holds. This proves the induction step, and since the base case holds, $P(n)$ is true for all $n \geq 1$ by induction. ∎