Math 270 - Basic Discrete Mathematics
Practice Quiz on Section 5.4
Solutions

Directions: Answer the problem given below.

1. Let $a_1, a_2, a_3, \ldots$ be the sequence defined as follows:

$$a_1 = 2, a_2 = 20, \text{ and } a_n = 6a_{n-1} - 8a_{n-2} \text{ for all } n \geq 3.$$

Prove that for all integers $n \geq 1$, $a_n = 2 \cdot 4^n - 3 \cdot 2^n$.

Proof: Let $P(n) =$ "$a_n = 2\cdot 4^n - 3\cdot 2^n$". We prove $P(n)$ holds for all $n \geq 1$ by strong induction.

Base cases $(n=1,2)$: When $n=1$, $a_1 = 2$ and $2\cdot 4^1 - 3\cdot 2^1 = 8-6=2$, so $P(1)$ holds. When $n=2$, $a_2 = 20$ and $2\cdot 4^2 - 3\cdot 2^2 = 32-12=20$, so $P(2)$ holds.

Induction Step: Let $k \geq 2$ and suppose $P(1), P(2), \ldots, P(k)$ all hold. We must show $P(k+1)$ holds, i.e. that $a_{k+1} = 2\cdot 4^{k+1} - 3\cdot 2^{k+1}$.

Since $k \geq 2$, $k+1 \geq 3$, so

$$\begin{aligned} a_{k+1} &= 6a_k - 8a_{k-1} \\ &= 6(2\cdot 4^k - 3\cdot 2^k) - 8(2\cdot 4^{k-1} - 3\cdot 2^{k-1}) \\ &= 12\cdot 4^k - 18\cdot 2^k - 16\cdot 4^{k-1} + 24\cdot 2^{k-1} \\ &= 3\cdot 4^{k+1} - 9\cdot 2^{k+1} - 4^{k+1} + 6\cdot 2^{k+1} \\ &= 2\cdot 4^{k+1} - 3\cdot 2^{k+1}. \end{aligned}$$

Thus, $P(k+1)$ holds, and the induction step is proven.

Since the base case and induction step hold, $P(n)$ is true for all $n \geq 1$ by strong induction. ∎