Math 270 - Basic Discrete Mathematics
Practice Quiz on Section 5.6

Solutions

Directions: Answer the problems given below.

1. Find the first four terms of the sequence defined recursively as

$$a_1 = 1, a_2 = 2, \text{ and } a_k = a_{k-1} + a_{k-2} + 2 \text{ for all } k \geq 3.$$

$a_1 = 1$, $a_2 = 2$, $a_3 = a_2 + a_1 + 2 = 2 + 1 + 2 = 5$,

and $a_4 = a_3 + a_2 + 2 = 5 + 2 + 2 = 9$.

2. Recall that F_n denotes the nth Fibonacci number, defined in Example 5.6.6. Use mathematical induction to prove that for all integers $n \geq 0$,

$$F_{n+2}F_n - F_{n+1}^2 = (-1)^n.$$

(*Hint: Express* F_{n+2} *and one* F_{n+1} *above using the Fibonacci recurrence.*)

Proof: Let $P(n) =$ "$F_{n+2}F_n - F_{n+1}^2 = (-1)^n$". We'll prove $P(n)$ holds for all $n \geq 0$ by induction.

Base Case ($n=0$): $F_2F_0 - F_1^2 = 2 \cdot 1 - 1^2 = 1 = (-1)^0$, so $P(0)$ holds.

Inductive Step: Let $k \geq 0$ and suppose $P(k)$ holds. We must show $P(k+1)$ holds: consider the LHS of $P(k+1)$:

$$\begin{aligned} F_{k+3}F_{k+1} - F_{k+2}^2 &= (F_{k+2} + F_{k+1})F_{k+1} - F_{k+2}(F_{k+1} + F_k) \quad (\text{since } k+2 \geq 2,\ k+3 \geq 2) \\ &= F_{k+2}F_{k+1} + F_{k+1}^2 - F_{k+2}F_{k+1} - F_{k+2}F_k \\ &= -(F_{k+2}F_k - F_{k+1}^2) \\ &= -(-1)^k \quad (\text{by I.H}) \\ &= (-1)^{k+1}, \end{aligned}$$

so $P(k+1)$ holds. This proves the inductive step,

and since the base case holds, $P(n)$ is true for all $n \geq 0$ by induction. ∎