

Math 270 - Basic Discrete Mathematics  
Practice Quiz on Section 5.7

*Solutions*

**Directions:** Answer the problem given below.

1. Let  $x_1, x_2, x_3, \dots$  be the sequence defined recursively as

$$x_1 = 1 \text{ and for all } k \geq 2, x_k = x_{k-1} + 2k + 1.$$

Find (but do not prove) an explicit formula for this sequence.

$$x_1 = 1$$

$$x_2 = x_1 + 2(2) + 1 = 1 + 2(2) + 1$$

$$\begin{aligned} x_3 &= x_2 + 2(3) + 1 = 1 + 2(2) + 1 + 2(3) + 1 \\ &= 3 + 2(2) + 2(3) \end{aligned}$$

$$\begin{aligned} x_4 &= x_3 + 2(4) + 1 = 3 + 2(2) + 2(3) + 2(4) + 1 \\ &= 4 + 2(2) + 2(3) + 2(4) \end{aligned}$$

⋮

$$x_n = n + 2(2) + 2(3) + \dots + 2(n)$$

$$= n + 2(2 + 3 + \dots + n)$$

$$= n + 2(1 + 2 + 3 + \dots + n - 1)$$

$$= n + 2\left(\frac{n(n+1)}{2} - 1\right)$$

$$= n + n(n+1) - 2, \text{ so}$$

$$x_n = n^2 + 2n - 2.$$