

Math 270 - Basic Discrete Mathematics  
Practice Quiz on Section 5.8

Solutions

Directions: Answer the problems given below.

1. Find an explicit formula for the sequence  $a_1, a_2, a_3, \dots$  given by

$$a_1 = 1, a_2 = 2, \text{ and } a_k = 3a_{k-1} - 2a_{k-2} \text{ for all } k \geq 3.$$

2<sup>nd</sup> order homogeneous linear recurrence relation (!)

The characteristic equation is

$$t^2 - 3t + 2 = (t-2)(t-1),$$

so  $\exists$  reals  $C, D$  such that

$$a_n = C \cdot 2^n + D \cdot 1^n$$

$$n=1 : 1 = 2C + D \rightsquigarrow D = 1 - 2C \quad \xrightarrow{D=0}$$

$$n=2 : 2 = 4C + D$$

$$\xrightarrow{2=2C+1} 2=2C+1 \rightsquigarrow C = \frac{1}{2}$$

So,

$$a_n = \frac{1}{2} \cdot 2^n.$$