

Math 270 - Basic Discrete Mathematics
Practice Quiz on Section 5.9

Solutions

Directions: Answer the problems given below.

1. Define a set S of integers recursively as follows:

I. Base: $0 \in S$.

II. Recursion: if $k \in S$, then

II(a) $k + 4 \in S$

II(b) $k - 4 \in S$

III. Restriction: Nothing is in S other than objects defined in I, II above.

Use structural induction to prove that every integer $n \in S$ is divisible by 4.

Proof: let $P(n) = "n \text{ is divisible by } 4"$: we prove $P(n)$ holds for all $n \in S$ by structural induction.

Base: The only element of the base is $0 = 0 \cdot 4$, so $P(0)$ holds.

Inductive Step: let $k \in S$ and suppose $P(k)$ holds, so k is a multiple of 4, say, $k = 4a$ for some $a \in \mathbb{Z}$. The recursion for S has two rules: when II(a) is applied the result is $k+4 = 4(a+1)$ $P(k+4)$ holds. When II(b) is applied the result is $k-4 = 4(a-1)$, so $P(k-4)$ holds. Thus, we see that the inductive step holds.

Since there are no integers in S other than those obtained from the base and the recursion for S , every integer in n is a multiple of 4. \Rightarrow