Math 270 - Basic Discrete Mathematics
Practice Quiz on Section 9.3

Solutions

Directions: Answer the problems given below.

1. A state decides that its license plate numbers will each consist of 7 characters: a nonzero digit from 1-9, followed by three upper-case Roman letters (from A-Z), followed by three more digits from 0-9. How many license plates contain a repeated letter? (*Hint: How many don't contain a repeated letter?*)

license plates : $9 \cdot 26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 158,184,000$

with no repeated letter : $9 \cdot 26 \cdot 25 \cdot 24 \cdot 10 \cdot 10 \cdot 10 = 140,400,000$,

so # with a repeated letter is their difference,

$$\boxed{17,784,000.}$$

2. How many integers from 1 to 2000 are divisible by 4 or by 5?

Let $A = \{a \in \mathbb{Z} \mid 1 \le a \le 2000,\ 4 \mid a\}$

$B = \{b \in \mathbb{Z} \mid 1 \le b \le 2000,\ 5 \mid b\}$:

we want $|A \cup B|$, which is $|A| + |B| - |A \cap B|$ by inclusion-exclusion. Observing that

$|A| = \left\lfloor \frac{2000}{4} \right\rfloor = 500$, $|B| = \left\lfloor \frac{2000}{5} \right\rfloor = 400$,

and $|A \cap B| = \left\lfloor \frac{2000}{20} \right\rfloor = 100$, we get $\boxed{|A \cup B| = 800.}$

Since 4, 5 have no prime factors in common,

$A \cap B = \{x \in \mathbb{Z} \mid 1 \le x \le 2000,\ 4 \mid x,\ 5 \mid x\}$

$= \{x \in \mathbb{Z} \mid 1 \le x \le 2000,\ 20 \mid x\}$