Math 270 Basic Discrete Math
Practice Test 1
Sections 1.4, 1.2, 2.1, 2.2, 3.1, 3.2, 3.3, 4.1

Name: (Please Print) Solutions

Directions: Answer the problems below. You may use scientific (non-graphing) calculators, but no other electronic devices. Show all work.

1. Determine whether the logical expressions $(p \to q) \wedge (p \to r)$ and $p \to (q \wedge r)$ are logically equivalent or not. Justify your answer.

Solution: We construct a truth table to check:

p	q	r	$p \to q$	$p \to r$	$(p \to q) \wedge (p \to r)$	$q \wedge r$	$p \to (q \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	F	F	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	F	T
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

Identical

Since both statement forms take the same truth values for every combination of their variables' truth values, $\boxed{\text{yes, they are logically equivalent.}}$

2. Answer parts a. and b. below. In both parts your answers may be sentences, and you do not need to determine whether the given statements are true or false.

a. Write negations for each of the following statements.

(i) Math 270 is relaxing and enjoyable.

Math 270 is not relaxing or is not enjoyable.

(ii) Every multiple of five is an odd number.

There is a multiple of five which is even.

(iii) There exists a real number x such that $x^7 = -20$.

For all real numbers x, $x^7 \neq -20$.

b. Write contrapositives for each of the following universal conditionals.

(i) For all squares S, if S has side length ℓ then S has area ℓ^2.

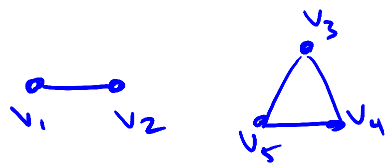
For all squares S, if S doesn't have area ℓ^2, S's side length is not ℓ.

(ii) For all integers m and n, if mn is even then m is even or n is even.

For all integers m and n, if m is odd and n is odd then mn is odd.

3. Provide short answers for parts a.-d.

a. Draw a graph G with 5 vertices with degrees $1, 1, 2, 2, 2$.



b. State the definition for an integer n to be *composite*.

n is composite if there exist integers r, s such that

$n = r \cdot s$, $1 < r < n$ and $1 < s < n$.

c. Let P be the set of all prime divisors of the number 2400: express P in set-roster notation.

$2400 = 24 \cdot 100 = 3 \cdot 8 \cdot 4 \cdot 25 = 2^5 \cdot 3 \cdot 5^2$,

so $P = \{2, 3, 5\}$.

d. What are De Morgan's Laws?

That

$$\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$$

and

$$\sim(p \vee q) \equiv (\sim p) \wedge (\sim q).$$

4. Let S be the set of students at CSUSM, G be the set of all video games ever made, and let $P(s, g)$ be the predicate "student s has played game g".

a. Rewrite each of the following as a sentence without using the symbols $\forall$ or $\exists$, and without using variables.

i. $\forall s \in S, \exists g \in G$ such that $P(s, g)$.

Every student at CSUSM has played a video game.

ii. $\exists s \in S$ such that $\forall g \in G, \sim P(s, g)$.

There is a student at CSUSM who has played every video game.

iii. $\forall s \in S, \quad P(s, \textit{Mario Kart 8 Deluxe}) \vee P(s, \textit{Minesweeper})$.

Every student at CSUSM has played Mario Kart 8 Deluxe or Minesweeper.

b. Rewrite each of the following sentences symbolically, using variables along with the symbols $\forall$ and $\exists$ and the predicate $P(s, g)$.

i. There is a game that every student at CSUSM has played.

$\exists g \in G$ such that $\forall s \in S, \ P(s, g)$.

ii. There is a game Student A has played that Student B has not played.

$\exists g \in G$ such that $P(A, g) \wedge (\sim P(B, g))$.

iii. Student A has played every game that Student B has played.

$\forall g \in G, \ P(B, g) \rightarrow P(A, g)$.

5. Answer parts a. and b. below.

a. Prove the following statement: There exist integers a and b such that $(a+b)^2 = a^2 + b^2$.

Proof: Let $a = 0$ and $b = 0$: then $a, b \in \mathbb{Z}$, and

$(a+b)^2 = 0^2 = 0$ and $a^2 + b^2 = 0^2 + 0^2 = 0$,

so $(a+b)^2 = a^2 + b^2$. $\blacksquare$

b. Find a counterexample that shows the following statement is false: For all integers n, if n is odd then $\dfrac{n+1}{2}$ is also odd.

Let $n = 3$: then n is odd (as $n = 2(1) + 1$)

but $\dfrac{n+1}{2} = \dfrac{4}{2} = 2$ is even.