Math 270 Basic Discrete Math
Practice Test 2
Sections 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8, 4.9

Name: (Please Print) Solutions

Directions: Answer the problems below. You may use scientific (non-graphing) calculators, but no other electronic devices. Show all work.

1. Prove that for all integers n, if n is odd then $(n+2)^2$ is odd.

Proof: Let $n \in \mathbb{Z}$ be arbitrary and suppose n is odd.

Then by definition of odd there exists a $k \in \mathbb{Z}$ such that $n = 2k+1$.

Then

$$(n+2)^2 = (2k+3)^2 = 4k^2 + 12k + 9 = 2(2k^2 + 6k + 4) + 1.$$

Since $k \in \mathbb{Z}$, $2k^2 + 6k + 4 \in \mathbb{Z}$ by closure (products and sums of integers are integers), so $(n+2)^2$ is odd by definition. And since n was arbitrary, the proof is complete. ∎

2. Prove that for all real numbers r and s, if r and s are rational then their average $\frac{r+s}{2}$ is also rational.

Proof: Let $r, s \in \mathbb{R}$ be arbitrary and suppose both are rational.

Then $\exists\, a, b, c, d \in \mathbb{Z}$ such that $r = \frac{a}{b}$, $s = \frac{c}{d}$, $b \neq 0$, $d \neq 0$.

Therefore,

$$\frac{r+s}{2} = \frac{\frac{a}{b} + \frac{c}{d}}{2} = \frac{\frac{ad+bc}{bd}}{2} = \frac{ad+bc}{2bd}.$$

Since $a, b, c, d \in \mathbb{Z}$ we have $ad+bc \in \mathbb{Z}$ and $2bd \in \mathbb{Z}$ by closure, and since $b \neq 0$, $d \neq 0$, we have that $2bd \neq 0$ by the Zero Product Property. Therefore $\frac{r+s}{2}$ is rational by definition, and the result follows as r, s were arbitrary. $\blacksquare$

3. Answer parts a.-d. by circling *ANY AND ALL* correct answers. (There may be multiple correct answers.)

a. Which of the following are true?

50 div 7 = 1	25 mod 6 = 4
15 div 8 = 1	45 div 9 = 5

b. A simple graph G has 6 vertices: which of the following can be the degrees of its vertices?

1,1,1,1,1,1	2,1,2,1,2,3.
6,6,6,6,6,6	1,1,2,2,2,2.

c. Suppose that a and b are integers, and that $2|a$ and $3|b$. Which of the following are true (no matter which a, b are chosen)?

$2\|(a-4b)$	$5\|(a+b)$
$6\|(3a+2b)$	$18\|(a^2b^3)$

d. Which of the following are true?

$\lfloor -0.5 \rfloor = 0$	$\lceil \pi \rceil = 4$
If n is odd then $\left\lfloor \frac{n}{2} \right\rfloor = \frac{n+1}{2}$	For any real x, $\lceil x \rceil < x+1$.

4. Prove that if n is an integer, then the number $18n + 7$ is *not* a multiple of 9.

Proof: By way of contradiction, suppose $\exists n \in \mathbb{Z}$ such that $9 \mid (18n+7)$. Then $\exists k \in \mathbb{Z}$ such that $18n+7 = 9k$, so $7 = 9k - 18n = 9(k-2n)$. Since $k - 2n \in \mathbb{Z}$ by closure, therefore $9 \mid 7$. But this is a contradiction (to Thm 4.4.1) as $9 > 7$, so no such n exists. $\blacksquare$

5. Prove, using the definition of odd, that if n is any integer, then $n^2 + 5n + 1$ is odd. (*Hint: Consider the parity of* n.)

Proof: Let $n \in \mathbb{Z}$ be arbitrary. We consider two cases, depending on the parity of n.

Case 1 (n is even): If n is even, then $\exists k \in \mathbb{Z}$ such that $n = 2k$, so

$$n^2 + 5n + 1 = 4k^2 + 10k + 1 = 2(2k^2 + 5k) + 1.$$

Since $k \in \mathbb{Z}$, $2k^2 + 5k \in \mathbb{Z}$ by closure, so $n^2 + 5n + 1$ is odd.

Case 2: (n is odd) If n is odd, then $\exists k \in \mathbb{Z}$ such that $n = 2k+1$. Then

$$\begin{aligned} n^2 + 5n + 1 &= (4k^2 + 4k + 1) + 10k + 5 + 1 \\ &= (4k^2 + 14k + 6) + 1 \\ &= 2(2k^2 + 7k + 3) + 1. \end{aligned}$$

Since $k \in \mathbb{Z}$, $2k^2 + 7k + 3 \in \mathbb{Z}$ by closure, so $n^2 + 5n + 1$ is odd.

In either case, $n^2 + 5n + 1$ is odd, so the result holds as n was arbitrary. ∎